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Rank 1

Cracking the
JEE Main Code
360 out of 360
Kalpit Veerwal

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Class XI | XII

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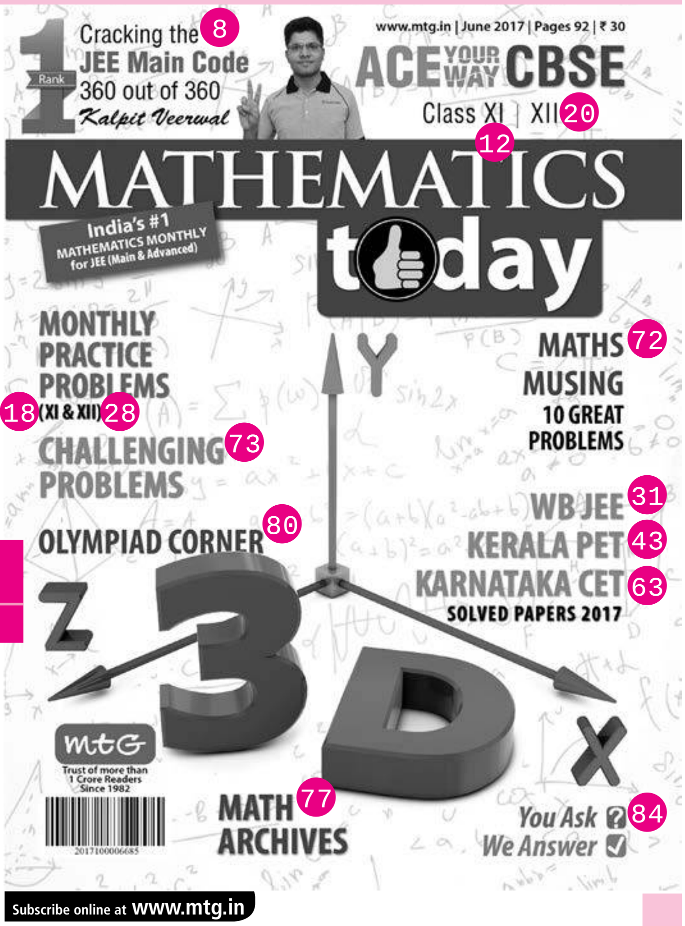
CONTENTS



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MtG Team Applauds

JEE (Main) 2017 Topper

*"It gives us immense pleasure to felicitate the achievement of our reader Kalpit Veerwal **(MTG Subscription Code PCM-102332)**. We feel proud that we could lay a brick for the foundation of his success. We are sharing his success story here so that it can inspire others to ace in exams."*

Kalpit Veerwal

Rank 1

- **MTG : Why did you appear for Engineering Entrance?**

  Kalpit : I had interest in studying Science and Mathematics, so wanted to appear for the entrance exams. Now, I want to opt for Computer Science Engineering.

- **MTG : What exams have you appeared for and what are your ranks in these exams?**

  Kalpit : I have appeared for JEE Main 2017 in which I got AIR 1 with 360 marks out of 360. I have also achieved ranks in exams like NSO-7th International rank, IMO-17th International rank (conducted by Science Olympiad Foundation, New Delhi), NTSE (Stage I-Rajasthan 1st rank and Stage II-cleared), KVPY, IJSO Stage II (Top 35), IJAO Stage II (Top 20) and IAO Stage II (Top 25).

- **MTG : How many hours in a day did you dedicate for the preparation of the examination?**

  Kalpit : Apart from studying in school and coaching, I used to spend 5-6 hours on self study.

- **MTG : Any extra coaching?**

  Kalpit : I pursued coaching from Resonance, Udaipur.

- **MTG : Did you appear for the offline JEE Main exam or the online one? What is the reason behind this choice of yours?**

  Kalpit : I appeared for the JEE Main offline exam as I believe that it is the traditional method. I was not very confident with the online option.

  Moreover, I was comfortable with the offline pattern as my coaching institute follows it. Therefore, I chose to take my exam in pen and paper pattern.

- **MTG : On which topic and chapters you laid more stress in each subject?**

  Kalpit : I focussed on complete syllabus, rather than selective study. Hence, gave equal weightage to all chapters of each subject.

- **MTG : How much time does one require for serious preparation of this exam?**

  Kalpit : I studied sincerely from class VIII but any kind of serious preparation from my side for JEE Main started from class XI. Every day after coming back from school, I studied regularly. I never bunked any of my classes and followed my teachers religiously. I used to wake up early in the morning to study.

> **"I was a 2 year subscriber of the MTG magazines : Physics For You, Chemistry Today and Mathematics Today and was really an avid reader of them. They were highly useful especially because of the good question bank and articles published in every issue. I also studied from NCERT books and solved JEE Main previous years question papers."**

- **MTG : Was there a difference in the preparation strategy during the last months of JEE Main considering that the board exams were also scheduled in this period? How did you manage the preparation for both?**

  Kalpit : As you know that board exams were scheduled in March and JEE Main 2017 was scheduled in April so I revised

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1st in C.G.
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AIR-223
Keshav Gupta
AIR-232
Ashish R. Nair
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Arooshi Verma
AIR-2463
Vinamra Bathwal
AIR-3070
Jheel Rathoo
AIR-3191
Shrirang Vaidya
AIR-3231
Pravartya Kr. Dewangan
AIR-3496
Harsh Prakash Gupta
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my complete syllabus of class XII boards in November and once again in February. When one month was left I revised the entire syllabus from NCERT.

- **MTG : Which books/magazines you read?**

  Kalpit : I read HC Verma, Resnik, Irodov, SL Loney and JD Lee.

- **MTG : In your words what are the components of an ideal preparation plan?**

  Kalpit : There are 3 main components of an ideal preparation plan : Focus, Practice and Dedication. Stick to the basics and master the speed with the help of regular tests.

- **MTG : What role did the following play in your success:**

  (a) Parents (b) Teachers and School

  Kalpit : (a) My parents took care of my health and kept me stress free.

  (b) My teachers and school were very supportive as they taught all the concepts and covered the syllabus.

- **MTG : What is your family background?**

  Kalpit : My father is a nurse in a government hospital. My mother is a teacher in government school and my brother is pursuing MBBS at AIIMS, Jodhpur.

- **MTG : How have MTG magazines helped you in your preparation?**

  Kalpit : I was a 2 year subscriber of MTG magazines : Physics For You, Chemistry Today and Mathematics Today and was really an avid reader of them. These magazines were highly useful especially because of the good question bank and articles published in every issue.

- **MTG : Was this your first attempt?**

  Kalpit : Yes, this was my first attempt.

- **MTG : What do you think is the secret of your success?**

  Kalpit : Consistency, dedication and hardwork is the secret. Proper preparation requires you to practice questions on regular basis.

- **MTG : How did you de-stress yourself during the preparation? What are your hobbies? How often could you pursue them?**

  Kalpit : I used to listen to music, play cricket and badminton to release the stress, in between my studies. I like listening to Coldplay and Linkin Park.

- **MTG : What do you feel is lacking in our education/ examination system? Is the examination system fair to the student?**

  Kalpit : Yes, it is fair.

- **MTG : Had you not been selected then what would have been your future plan?**

  Kalpit : I have not thought about it.

- **MTG : What advice would you like to give to our readers who are JEE aspirants?**

  Kalpit : I advise to all aspirants that continuous hardwork along with dedication and time management would give the desired results.

All the Best!☺☺

CLASS XI **Series 2**

ACE YOUR WAY CBSE

Relations and Functions

Important Formulae

- If A and B be two non-empty sets, then cartesian product of sets is
  $$A \times B = \{(x_i, y_i) : x_i \in A, y_i \in B\}$$
- $(x, y) = (p, q) \Leftrightarrow x = p, y = q$
- $A \times B = B \times A \Rightarrow A = B$
- If $n(A) = p$, $n(B) = q$, then $n(A \times B) = pq$
- $A \times A \times A = \{(a, b, c) : a, b, c \in A\}$ is called ordered triplet.
- $A \times B = \phi \Leftrightarrow A = \phi$ or $B = \phi$
- Subset of $X \times Y$ is called a relation from X to Y.
- If $n(X) = p$ and $n(Y) = q$ then the total number of relations from X to Y is 2^{pq}.
- The set of all first elements of the ordered pairs in relation R from set X to set Y is called the domain of the relation R.
- The set of all second elements of the ordered pairs in relation R from set X to Y is called the range of the relation R.
- The whole set B is called the co-domain of the relation R.
  - Range $\subseteq$ Co-domain
- If $R = \{(a, b) : a, b \in R\}$, then
  $R^{-1} = \{(b, a) : b, a \in R\}$
- A subset f of $X \times Y$ is called a function (or map or mapping) from X to Y iff for each $x \in X$, there exists a unique $y \in Y$ such that $(x, y) \in f$. It is written as $f : X \to Y$.
  - Set X is called domain and Set Y is called co-domain of the function f.
  - The set of elements of Y, which are assigned to the elements of X is called range of f.
- Algebra of real functions
  If $f : X \to R$ and $g : X \to R$, then
  - $(f + g)(x) = f(x) + g(x), \forall x \in X$
  - $(f - g)(x) = f(x) - g(x), \forall x \in X$
  - $(\alpha f)(x) = \alpha f(x), \forall x \in X$
  - $(fg)(x) = f(x)g(x), \forall x \in X$
  - $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0, x \in X$
- Function $\subseteq$ Relation $\subseteq$ Cartesian Product
- If A and B be two non-empty sets having n elements in common, then $A \times B$ and $B \times A$ have n^2 elements in common.

WORK IT OUT

VERY SHORT ANSWER TYPE

1. Let $A = \{1, 2\}$ and $B = \{3, 4, 5\}$. Find
   (i) $B \times A$ (ii) $A \times A \times A$

2. Let $P = \{x, y, z\}$ and $Q = \{3, 4\}$. Find the number of relations from P to Q.

3. Let f be the exponential function and g be the logarithmic function. Find $(fg)(1)$.

4. Let $f(x) = x^2$ and $g(x) = (3x + 2)$ be two real functions. Then, find $(f + g)(x)$.

5. Let $g = \{(1, 2), (2, 5), (3, 8), (4, 10), (5, 12), (6, 12)\}$ Is g a function? If yes, find its domain and range. If no, give reason.

SHORT ANSWER TYPE

6. Let $f : Z \to Z$, $g : Z \to Z$ be functions defined by $f = \{n, n^2) : n \in Z\}$ and $g = \{(n, |n|^2) : n \in Z\}$. Show that $f = g$.

7. The function $F(x) = \frac{9x}{5} + 32$ is the formula to convert $x°$ C to Fahrenheit units. Find
   (i) $F(0)$ (ii) $F(-10)$
   (iii) the value of x when $F(x) = 212$.

8. If $f(x) = \log\left(\frac{1+x}{1-x}\right)$, prove that $f\left(\frac{2x}{1+x^2}\right) = 2f(x)$.

9. Let R be a relation on the set of natural numbers N defined by $xRY \Leftrightarrow x + 2y = 41$; $\forall\ x, y \in N$. Find the domain and range of R.

10. If $P = \{a, b\}$ and $Q = \{x, y, z\}$, then show that $P \times Q \neq Q \times P$.

LONG ANSWER TYPE - I

11. Given $f(x) = \frac{1}{(1-x)}$, $g(x) = f\{f(x)\}$ and $h(x) = f[f\{f(x)\}]$. Then find the value of $f(x) \cdot g(x) \cdot h(x)$.

12. Find the domain of the function
$$y = \frac{1}{\log_{10}(1-x)} + \sqrt{x+2}$$

13. If $A \subseteq B$ and $C \subseteq D$, prove that $A \times C \subseteq B \times D$.

14. Let f be a real valued function defined by $f(x) = \frac{e^x - e^{-|x|}}{e^x + e^{|x|}}$, then find the range of f.

15. Consider the following :
   (i) $f : R \to R : f(x) = \log_e x$
   (ii) $g : R \to R : g(x) = \sqrt{x}$
   (iii) $h : A \to R : h(x) = \frac{1}{x^2 - 4}$, where $A = R - \{-2, 2\}$
   Which of them are functions? Also find their range, if they are functions.

LONG ANSWER TYPE - II

16. Find the domain and range of the real valued function $f(x)$ given by $f(x) = \frac{4-x}{x-4}$.

17. If $f : R \to R$ is defined by $f(x) = x^3 + 1$ and $g : R \to R$ is defined by $g(x) = x + 1$, then find
   (i) $f + g$ (ii) $f - g$ (iii) $f \cdot g$
   (iv) $\frac{f}{g}$ (v) $\alpha f\ (\alpha \in R)$

18. Find the domain and range of the function $f(x) = \frac{1}{2 - \sin 3x}$

19. Let $A = \{x \in N : x^2 - 5x + 6 = 0\}$, $B = \{x \in Z : 0 \le x < 2\}$ and $C = \{x \in N : x < 3\}$, then verify that:
   (i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$
   (ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

20. Find the domain of the function
$$f(x) = \sqrt{(\log_2(x))} + \sqrt{7x - x^2 - 6}$$

SOLUTIONS

1. (i) $B \times A = \{3, 4, 5\} \times \{1, 2\}$
$= \{(3, 1), (3, 2), (4, 1), (4, 2), (5, 1), (5, 2)\}$
(ii) $A \times A \times A = \{1, 2\} \times \{1, 2\} \times \{1, 2\}$
$= \{(1, 1, 1), (1, 1, 2), (1, 2, 1), (1, 2, 2), (2, 1, 1), (2, 1, 2), (2, 2, 1), (2, 2, 2)\}$

2. Given, $P = \{x, y, z\}$ and $Q = \{3, 4\}$
$\therefore$ $n(P) = 3$ and $n(Q) = 2$
$\therefore$ $n(P \times Q) = 3 \cdot 2 = 6$
Total number of relations from P to Q = number of subsets of $P \times Q = 2^6 = 64$.

3. We have, $f : R \to R$ given by $f(x) = e^x$ and $g : R^+ \to R$ given by $g(x) = \log_e x$
Domain $(f) \cap$ Domain $(g) = R \cap R^+ = R^+$.
$\therefore$ $fg : R^+ \to R$ is given by $(fg)(x) = f(x)\, g(x) = e^x \cdot \log_e x$.
Now, $(fg)(1) = f(1)g(1) = e^1 \times \log_e 1 = e \times 0 = 0$.

4. We have $(f + g)(x) = f(x) + g(x) = x^2 + (3x + 2)$

5. Yes, dom $(g) = \{1, 2, 3, 4, 5, 6\}$,
range $(g) = \{2, 5, 8, 10, 12\}$

6. Given, domain of $f = Z$ and domain of $g = Z$

Hence, domain f = domain $g = Z$ ...(1)

Also, $f(n) = n^2$, for all $n \in Z$ and $g(n) = |n|^2 = n^2$ for all $n \in Z$

Hence, $f(n) = g(n)$ for all $n \in Z$ ...(2)

From (1) and (2), we have $f = g$.

7. $F(x) = \frac{9x}{5} + 32$ (given)

(i) $F(0) = \left(\frac{9\times 0}{5} + 32\right) = 32 \Rightarrow F(0) = 32$

(ii) $F(-10) = \left\{\frac{9\times(-10)}{5} + 32\right\} = 14 \Rightarrow F(-10) = 14$

(iii) $F(x) = 212 \Leftrightarrow \frac{9x}{5} + 32 = 212$

$\Leftrightarrow 9x = (5 \times 180) \Leftrightarrow x = 100$

8. Given, $f(x) = \log\left(\frac{1+x}{1-x}\right)$

$$\therefore f\left(\frac{2x}{1+x^2}\right) = \log\left(\frac{1+\frac{2x}{1+x^2}}{1-\frac{2x}{1+x^2}}\right) = \log\left(\frac{1+x^2+2x}{1+x^2-2x}\right)$$

$$= \log\left(\frac{1+x}{1-x}\right)^2 = 2\log\left(\frac{1+x}{1-x}\right) = 2f(x).$$

9. We have, $y = \frac{41-x}{2} \in N$

Clearly $x = 1, 3, 5, 7, \ldots, 39$

$\therefore$ Domain $R = \{x : (x, y) \in R;\ x + 2y = 41\}$

$= \{1, 3, 5, 7, \ldots, 39\}$

= set of odd natural numbers less than 40.

Now, y can be only those natural numbers for which $x \in N$ *i.e.*, $x = 41 - 2y \in N$.

Clearly, $y = 1, 2, 3, \ldots, 20$.

$\therefore$ Range of $R = \{y : x + 2y = 41\} = \{1, 2, 3, \ldots, 20\}$

= set of natural numbers less than 21.

10. We have, $P = \{a, b\}$ and $Q = \{x, y, z\}$

Now, $P \times Q = \{(a, x), (a, y), (a, z), (b, x), (b, y), (b, z)\}$

$Q \times P = \{(x, a), (x, b), (y, a), (y, b), (z, a), (z, b)\}$

$\therefore P \times Q \neq Q \times P$.

11. Given, $g(x) = f\{f(x)\}$

$$= f\left(\frac{1}{1-x}\right) = \frac{1}{1-\frac{1}{1-x}} = -\left(\frac{1-x}{x}\right)$$

and $h(x) = f[f\{f(x)\}] = f\{g(x)\} = f\left(-\left(\frac{1-x}{x}\right)\right)$

$$= \frac{1}{1+\frac{1-x}{x}} = x$$

$$\therefore f(x)\cdot g(x)\cdot h(x) = \frac{1}{1-x}\left\{-\left(\frac{1-x}{x}\right)\right\}\cdot x = -1$$

12. For y to be defined

(i) $\log_{10}(1 - x)$ must be defined $\Rightarrow 1 - x > 0 \Rightarrow x < 1$

(ii) $\log_{10}(1 - x) \neq 0 \Rightarrow 1 - x \neq 10^0 \Rightarrow 1 - x \neq 1 \Rightarrow x \neq 0$

(iii) $x + 2 \geq 0 \Rightarrow x \geq -2$

From (i), (ii) and (iii), we get $-2 \leq x < 1$ and $x \neq 0$

$\therefore -2 \leq x < 0$ or $0 < x < 1$

Hence domain $= [-2, 0) \cup (0, 1)$.

13. Let (a, b) be an arbitrary element of $A \times C$. Then,

$(a, b) \in A \times C$

$\Rightarrow a \in A$ and $b \in C$

$\Rightarrow a \in B$ and $b \in D$ $[\because A \subseteq B$ and $C \subseteq D]$

$\Rightarrow (a, b) \in B \times D$

Thus, $(a, b) \in A \times C$

$\Rightarrow (a, b) \in B \times D$ for all $(a, b) \in (A \times C)$.

$\therefore A \times C \subseteq B \times D$

14. Let $y = f(x) = \frac{e^x - e^{-|x|}}{e^x + e^{|x|}}$

If $x \geq 0$ then $y = \frac{e^{2x}-1}{2e^{2x}}$

$\Rightarrow e^{2x} = \frac{1}{1-2y} \geq 1$ $(\because x \geq 0)$

$\Rightarrow \frac{1}{1-2y} - 1 \geq 0 \Rightarrow \frac{y}{1-2y} \geq 0$ or $\frac{y}{2y-1} \leq 0$

$\therefore 0 \leq y < \frac{1}{2} \Rightarrow y \in \left[0, \frac{1}{2}\right)$

15. f and g are not functions as they are not defined for negative values of x. But h is a function.

$\therefore$ For range of h, Let $y = h(x) = \frac{1}{x^2-4}$

$\Rightarrow x^2 - 4 = \frac{1}{y} \Rightarrow x^2 = 4 + \frac{1}{y} \Rightarrow x = \sqrt{\frac{4y+1}{y}}$

Hence, range of $h = \left(-\infty, \frac{-1}{4}\right] \cup (0, \infty)$

MPP-2 CLASS XII ANSWER KEY

1.	(c)	**2.**	(b)	**3.**	(b)	**4.**	(d)	**5.**	(c)
6.	(c)	**7.**	(a,b)	**8.**	(b,c)	**9.**	(b,c)	**10.**	(b)
11.	(a,b)	**12.**	(a)	**13.**	(a,b,d)	**14.**	(d)	**15.**	(b)
16.	(d)	**17.**	(2)	**18.**	(3)	**19.**	(2)	**20.**	(7)

16. We have, $f(x)=\dfrac{4-x}{x-4}$.

Domain of f : We observe that $f(x)$ is defined for all x except at $x = 4$. At $x = 4$, $f(x)$ takes the indeterminate form $\frac{0}{0}$. Therefore, Domain $(f) = R - \{4\}$.

Range of f : For any $x \in$ Domain (f) *i.e.* for any $x \neq 4$, we have

$$f(x)=\frac{4-x}{x-4}=\frac{-(x-4)}{x-4}=-1.$$

$\therefore$ Range $(f) = \{-1\}$.

17. (i) $f+g: R \to R$ is defined by

$(f + g)(x) = f(x) + g(x) = x^3 + 1 + x + 1 = x^3 + x + 2$

(ii) $f - g : R \to R$ is defined by

$(f - g)(x) = f(x) - g(x) = x^3 + 1 - x - 1 = x^3 - x$

(iii) $f \cdot g : R \to R$ is defined by

$(fg)(x) = f(x)\, g(x) = (x^3 + 1)(x + 1) = x^4 + x^3 + x + 1$

(iv) $\dfrac{f}{g} : R - \{-1\} \to R$ is defined by

$$\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}=\frac{x^3+1}{x+1}=\frac{(x+1)(x^2-x+1)}{x+1}=x^2-x+1$$

(v) $\alpha f : R \to R$ is defined by

$(\alpha f)(x) = \alpha\, f(x) = \alpha\,(x^3 + 1) = \alpha\, x^3 + \alpha$

18. We have, $f(x)=\dfrac{1}{2-\sin 3x}$

Domain of f : We know that $-1 \leq \sin 3x \leq 1$ for all $x \in R$

$\Rightarrow -1 \leq -\sin 3x \leq 1$ for all $x \in R$

$\Rightarrow 1 \leq 2 - \sin 3x \leq 3$ for all $x \in R$

$\Rightarrow 2 - \sin 3x \neq 0$ for any $x \in R$

$\Rightarrow f(x)=\dfrac{1}{2-\sin 3x}$ is defined for all $x \in R$

Hence, domain $(f) = R$.

Range of f : $\because 1 \leq 2 - \sin 3x \leq 3$ for all $x \in R$

$\Rightarrow \dfrac{1}{3}\leq\dfrac{1}{2-\sin 3x}\leq 1$ for all $x \in R$

$\Rightarrow \dfrac{1}{3}\leq f(x)\leq 1$ for all $x \in R$

$\Rightarrow f(x) \in [1/3, 1]$

Hence, range $(f) = [1/3, 1]$

19. We have

$A = \{x \in N : x^2 - 5x + 6 = 0\} = \{2, 3\}$;

$B = \{x \in Z : 0 \leq x < 2\} = \{0, 1\}$ and

$C = \{x \in N : x < 3\} = \{1, 2\}$

$\therefore$ $A = \{2, 3\}$, $B = \{0, 1\}$ and $C = \{1, 2\}$

(i) $(B \cup C) = \{0, 1, 2\}$

$\therefore$ $A \times (B \cup C) = \{2, 3\} \times \{0, 1, 2\}$

$= \{(2, 0), (2, 1), (2, 2), (3, 0), (3, 1), (3, 2)\}$

$(A \times B) = \{2, 3\} \times \{0, 1\} = \{(2, 0), (2, 1), (3, 0), (3, 1)\}$

$(A \times C) = \{2, 3\} \times \{1, 2\} = \{(2, 1), (2, 2), (3, 1), (3, 2)\}$

$\therefore$ $(A \times B) \cup (A \times C) = \{(2, 0), (2, 1), (2, 2), (3, 0), (3, 1), (3, 2)\}$

Hence, $A \times (B \cup C) = (A \times B) \cup (A \times C)$

(ii) $(B \cap C) = \{0, 1\} = \{1\}$

$\therefore$ $A \times (B \cap C) = \{2, 3\} \times \{1\} = \{(2, 1), (3, 1)\}$

And,

$(A \times B) \cap (A \times C) = \{(2, 1), (3, 1)\}$

Hence, $A \times (B \cap C) = (A \times B) \cap (A \times C)$

20. We have, $f(x) = \sqrt{(\log_2(x))}+\sqrt{7x-x^2-6}$

$$=\sqrt{(\log_2(x))}+\sqrt{(1-x)(x-6)}$$

For $f(x)$ to be defined.

(i) $(\log_2(x)) \geq 0 \quad \Rightarrow \quad x \geq 2^0 \quad \Rightarrow \quad x \geq 1$

(ii) $(1 - x)(x - 6) \geq 0 \quad \Rightarrow \quad 1 \leq x \leq 6$

Therefore domain of $f = [1, 6]$.

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MPP-2 MONTHLY Practice Problems

Class XI

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Complex Numbers and Quadratic Equations

Total Marks : 80 **Time Taken : 60 Min.**

Only One Option Correct Type

1. $\left(\dfrac{1+i\sin\frac{\pi}{8}+\cos\frac{\pi}{8}}{1-i\sin\frac{\pi}{8}+\cos\frac{\pi}{8}}\right)^8$ equals

(a) 2^8 (b) 0
(c) – 1 (d) 1

2. If a, b, c are in G.P., then the equations $ax^2 + 2bx + c = 0$, $dx^2 + 2ex + f = 0$ have a common root if $\dfrac{d}{a}, \dfrac{e}{b}, \dfrac{f}{c}$ are in

(a) A.P. (b) G.P.
(c) H.P. (d) none of these

3. For positive integers n_1 and n_2 the value of the expression $(1+i)^{n_1} + (1+i^3)^{n_1} + (1+i^5)^{n_2} + (1+i^7)^{n_2}$ where $i = \sqrt{-1}$ is a real number iff

(a) $n_1 = n_2$ (b) $n_2 = n_2 - 1$
(c) $n_1 = n_2 + 1$ (d) $\forall\ n_1$ and n_2

4. If x, y, z are distinct positive reals such that $\dfrac{\log x}{y-z} = \dfrac{\log y}{z-x} = \dfrac{\log z}{x-y}$, then value of $x^x y^y z^z$ is

(a) 1 (b) 0
(c) – 1 (d) none of these

5. If the roots of the equation $bx^2 + cx + a = 0$ be imaginary, then for all real values of x, the expression $3b^2x^2 + 6bcx + 2c^2$ is

(a) less than $4ab$ (b) greater than $-4ab$
(c) less than $-4ab$ (d) greater than $4ab$

6. Solve for x :
$\log_{2x+3}(6x^2 + 23x + 21) + \log_{3x+7}(4x^2 + 12x + 9) = 4$

(a) -4 (b) – 2
(c) $-\dfrac{1}{4}$ (d) All of these

One or More Than One Option(s) Correct Type

7. $ABCD$ is a rhombus, its diagonals AC and BD intersect at the point R where $BD = 2AC$. Its points D and R represent the complex numbers $1 + i$ and $2 - i$ respectively, then the complex number represented by A is

(a) (3, – 1/2) or (1, – 1/2)
(b) (3, – 1/2) or (1, – 3/2)
(c) (– 1/2, – 3/2) or (– 3/2, – 1/2)
(d) None of these

8. $(1 + i)^5 + (1 - i)^5 =$

(a) – 8 (b) 8
(c) $2^{7/2}\cos\dfrac{5\pi}{4}$ (d) $-2^{7/2}\cos\dfrac{5\pi}{4}$

9. If α and β are non-real cube roots of unity and $x = a + b$, $y = a\alpha + b\beta$, $z = a\beta + b\alpha$, then

(a) $x + y + z = 1$
(b) $x + y + z = 0$
(c) $x^3 + y^3 + z^3 = 3(a^3 + b^3)$
(d) none of these

10. The equation whose roots are $\alpha\gamma + \beta\delta$ and $\alpha\delta + \gamma\beta$, if α, β are the roots of the equation $ax^2 + bx + c = 0$ and γ, δ are roots of the equation $a'x^2 + b'x + c' = 0$, is

(a) $aa'x^2 + bb'x + cc' = 0$
(b) $ax^2 + (a + a' + b')x + bc' = 0$
(c) $aa'x^2 - bb'x + cc' = 0$
(d) none of these

11. If A and B are the points $(3, -1)$ and $(2, 1)$ respectively, then the locus of the points $P(z)$, $z = x + yi$, $x, y \in R$, such that $|z - 3 + i| = |z - 2 - i|$ is
(a) a circle containing A and B
(b) P is equidistant from A and B
(c) right bisector of segment joining A and B
(d) none of these

12. The value of $x : |x^2 + 2x - 8| + x - 2 = 0$ is
(a) -5 (b) -2
(c) 2 (d) -3

13. If $a \in C$ be such that $|a| = 1$, then the equation $\left(\frac{1+iz}{1-iz}\right)^4 = a$ has all the roots
(a) real and distinct
(b) non-real
(c) two real and two non-real
(d) none of these

Comprehension Type

Let α, β be the roots of the equation $6x^2 + 6px + p^2 = 0$, where p is a real number.

14. If both α and β are greater than 2, then
(a) $p < -4$
(b) $p < -2\sqrt{6}$
(c) $p < -6 - 2\sqrt{6}$
(d) none of these

15. The equation whose roots are $(\alpha + \beta)^2$ and $(\alpha - \beta)^2$ is
(a) $3x^2 + 4p^2x + p^4 = 0$
(b) $3x^2 - 4p^2x + p^4 = 0$
(c) $3x^2 - 4p^2x - p^4 = 0$
(d) none of these

Matrix Match Type

16. Match the following :

	Column I		Column II
P.	If a and b are positive numbers and $\log \frac{a+b}{2} = \frac{1}{2}(\log a + \log b)$ then $\frac{a}{b}$ is equal to	1.	$3\sqrt{3}$
Q.	Let $A(2 + 0i)$, $B(-1 + \sqrt{3}i)$ and $C(-1 - \sqrt{3}i)$ be the vertices of ΔABC. Then, $6 \sin A$ is equal to	2.	1
R.	If one root of the equation $(x - 1)(7 - x) = \lambda$ is three times the other, then λ =	3.	5
S.	Conjugate of the complex number $-\frac{7}{2} - \frac{3\sqrt{3}i}{2}$ is	4.	$-2 + 3\omega$

	P	Q	R	S
(a)	4	2	3	1
(b)	3	4	1	2
(c)	1	4	3	2
(d)	2	1	3	4

Integer Answer Type

17. The number of values of x in the interval $[0, 3\pi]$ satisfying the equation $2\sin^2 x + 5\sin x - 3 = 0$ is

18. The least integral value of k for which $(k - 2)x^2 + 8x + k + 4 > 0$ for all $x \in R$, is

19. If α, β are non-real cube roots of unity then $(1 + \alpha)(1 + \beta)(1 + \alpha^2)(1 + \beta^2)(1 + \alpha^4)(1 + \beta^4)$....upto $2n$ factors is equal to

20. If α be non real cube root of unity, then $\left|\sqrt{\alpha}\right|$ equals

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Keys are published in this issue. Search now! ☺

SELF CHECK

No. of questions attempted ……
No. of questions correct ……
Marks scored in percentage ……

Check your score! If your score is

> 90%	EXCELLENT WORK !	You are well prepared to take the challenge of final exam.
90-75%	GOOD WORK !	You can score good in the final exam.
74-60%	SATISFACTORY !	You need to score more next time.
< 60%	NOT SATISFACTORY!	Revise thoroughly and strengthen your concepts.

CLASS XII Series 2

ACE YOUR WAY CBSE

Inverse Trigonometric Functions

Important Formulae

Functions	Domain	Range
$\sin^{-1}x$	$[-1, 1]$	$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$
$\cos^{-1}x$	$[-1, 1]$	$[0, \pi]$
$\tan^{-1}x$	$(-\infty, \infty)$	$\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

Functions	Domain	Range
$\cot^{-1}x$	$(-\infty, \infty)$	$(0, \pi)$
$\text{cosec}^{-1}x$	$(-\infty, -1] \cup [1, \infty)$	$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$
$\sec^{-1}x$	$(-\infty, -1] \cup [1, \infty)$	$[0, \pi] - \left\{\frac{\pi}{2}\right\}$

PROPERTIES OF INVERSE TRIGONOMETRIC FUNCTIONS

1.
- $\sin^{-1}(\sin x) = x, \ \forall \ x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
- $\cos^{-1}(\cos x) = x, \ \forall \ x \in [0, \pi]$
- $\tan^{-1}(\tan x) = x, \ \forall \ x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
- $\cot^{-1}(\cot x) = x, \ \forall \ x \in (0, \pi)$
- $\sec^{-1}(\sec x) = x, \ \forall \ x \in [0, \pi] - \left\{\frac{\pi}{2}\right\}$
- $\text{cosec}^{-1}(\text{cosec}\, x) = x, \ \forall \ x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

2.
- $\sin(\sin^{-1}x) = x, \ \forall \ x \in [-1, 1]$
- $\cos(\cos^{-1}x) = x, \ \forall \ x \in [-1, 1]$
- $\tan(\tan^{-1}x) = x, \ \forall \ x \in R$
- $\cot(\cot^{-1}x) = x, \ \forall \ x \in R$
- $\sec(\sec^{-1}x) = x, \ \forall \ x \in R - (-1, 1)$
- $\text{cosec}(\text{cosec}^{-1}x) = x, \ \forall \ x \in R - (-1, 1)$

3.
- $\sin^{-1}(-x) = -\sin^{-1}x, \ \forall \ x \in [-1, 1]$
- $\cot^{-1}(-x) = \pi - \cot^{-1}x, \ \forall \ x \in R$
- $\cos^{-1}(-x) = \pi - \cos^{-1}x, \ \forall \ x \in [-1, 1]$
- $\sec^{-1}(-x) = \pi - \sec^{-1}x, \ \forall \ x \in (-\infty, -1] \cup [1, \infty)$
- $\tan^{-1}(-x) = -\tan^{-1}x, \ \forall \ x \in R$
- $\text{cosec}^{-1}(-x) = -\text{cosec}^{-1}x, \ \forall \ x \in (-\infty, -1] \cup [1, \infty)$

4.	• $\sin^{-1}(1/x) = \text{cosec}^{-1}x, \forall\, x \in (-\infty, -1] \cup [1, \infty)$	5.	• $\sin^{-1}x + \cos^{-1}x = \pi/2, \forall\, x \in [-1, 1]$
	• $\cos^{-1}(1/x) = \sec^{-1}x, \forall\, x \in (-\infty, -1] \cup [1, \infty)$		• $\tan^{-1}x + \cot^{-1}x = \pi/2, \forall\, x \in R$
	• $\tan^{-1}(1/x) = \begin{cases} \cot^{-1}x &, \text{ for } x > 0 \\ -\pi + \cot^{-1}x &, \text{ for } x < 0 \end{cases}$		• $\sec^{-1}x + \text{cosec}^{-1}x = \pi/2, \forall\, x \in (-\infty, -1] \cup [1, \infty)$

6.	• $\tan^{-1}x + \tan^{-1}y = \begin{cases} \tan^{-1}\left(\dfrac{x+y}{1-xy}\right) &, \text{ if } x > 0,\ y > 0,\ xy < 1 \\ \pi + \tan^{-1}\left(\dfrac{x+y}{1-xy}\right) &, \text{ if } x > 0,\ y > 0 \text{ and } xy > 1 \\ -\pi + \tan^{-1}\left(\dfrac{x+y}{1-xy}\right) &, \text{ if } x < 0,\ y < 0 \text{ and } xy > 1 \end{cases}$
	• $\tan^{-1}x - \tan^{-1}y = \begin{cases} \tan^{-1}\left(\dfrac{x-y}{1+xy}\right) &, \text{ if } x > 0,\ y < 0,\ xy > -1 \\ \pi + \tan^{-1}\left(\dfrac{x-y}{1+xy}\right) &, \text{ if } x > 0,\ y < 0 \text{ and } xy < -1 \\ -\pi + \tan^{-1}\left(\dfrac{x-y}{1+xy}\right) &, \text{ if } x < 0,\ y > 0 \text{ and } xy < -1 \end{cases}$
7.	• $\sin^{-1}x + \sin^{-1}y = \begin{cases} \sin^{-1}\left\{x\sqrt{1-y^2} + y\sqrt{1-x^2}\right\} &, \text{ if } -1 \le x,\ y \le 1 \text{ and } x^2 + y^2 \le 1 \\ & \text{or} \\ & \text{if } xy < 0 \text{ and } x^2 + y^2 > 1 \\ \pi - \sin^{-1}\left\{x\sqrt{1-y^2} + y\sqrt{1-x^2}\right\} &, \text{ if } 0 < x,\ y \le 1 \text{ and } x^2 + y^2 > 1 \\ -\pi - \sin^{-1}\left\{x\sqrt{1-y^2} + y\sqrt{1-x^2}\right\} &, \text{ if } -1 \le x,\ y < 0 \text{ and } x^2 + y^2 \ge 1 \end{cases}$
	• $\sin^{-1}x - \sin^{-1}y = \begin{cases} \sin^{-1}\left\{x\sqrt{1-y^2} - y\sqrt{1-x^2}\right\} &, \text{ if } -1 \le x,\ y \le 1 \text{ and } x^2 + y^2 \le 1 \\ & \text{or} \\ & \text{if } xy > 0 \text{ and } x^2 + y^2 > 1 \\ \pi - \sin^{-1}\left\{x\sqrt{1-y^2} - y\sqrt{1-x^2}\right\} &, \text{ if } 0 < x \le 1,\ -1 \le y \le 0 \text{ and } x^2 + y^2 > 1 \\ -\pi - \sin^{-1}\left\{x\sqrt{1-y^2} - y\sqrt{1-x^2}\right\} &, \text{ if } -1 \le x < 0,\ 0 < y \le 1 \text{ and } x^2 + y^2 > 1 \end{cases}$
8.	• $\cos^{-1}x + \cos^{-1}y = \begin{cases} \cos^{-1}\left\{xy - \sqrt{1-x^2}\sqrt{1-y^2}\right\} &, \text{ if } -1 \le x,\ y \le 1 \text{ and } x + y \ge 0 \\ 2\pi - \cos^{-1}\left\{xy - \sqrt{1-x^2}\sqrt{1-y^2}\right\} &, \text{ if } -1 \le x,\ y \le 1 \text{ and } x + y \le 0 \end{cases}$
	• $\cos^{-1}x - \cos^{-1}y = \begin{cases} \cos^{-1}\left\{xy + \sqrt{1-x^2}\sqrt{1-y^2}\right\} &, \text{ if } -1 \le x,\ y \le 1 \text{ and } x \le y \\ -\cos^{-1}\left\{xy + \sqrt{1-x^2}\sqrt{1-y^2}\right\} &, \text{ if } -1 \le y \le 0,\ 0 < x \le 1 \text{ and } x \ge y \end{cases}$

9.	• $2\sin^{-1}x=\begin{cases}\sin^{-1}(2x\sqrt{1-x^2}) & , \text{ if } -\frac{1}{\sqrt{2}}\le x\le\frac{1}{\sqrt{2}}\\ \pi-\sin^{-1}(2x\sqrt{1-x^2}), & \text{if } \frac{1}{\sqrt{2}}\le x\le 1\\ -\pi-\sin^{-1}(2x\sqrt{1-x^2}) & , \text{ if } -1\le x\le-\frac{1}{\sqrt{2}}\end{cases}$	
	• $3\sin^{-1}x=\begin{cases}\sin^{-1}(3x-4x^3) & , \text{ if } -\frac{1}{2}\le x\le\frac{1}{2}\\ \pi-\sin^{-1}(3x-4x^3) & , \text{ if } \frac{1}{2}< x\le 1\\ -\pi-\sin^{-1}(3x-4x^3), & \text{if } -1\le x<-\frac{1}{2}\end{cases}$	
10.	• $2\cos^{-1}x=\begin{cases}\cos^{-1}(2x^2-1), \text{ if } 0\le x\le 1\\ 2\pi-\cos^{-1}(2x^2-1), \text{if } -1\le x\le 0\end{cases}$	• $3\cos^{-1}x=\begin{cases}\cos^{-1}(4x^3-3x) & , \text{ if } \frac{1}{2}\le x\le 1\\ 2\pi-\cos^{-1}(4x^3-3x), & \text{if } -\frac{1}{2}\le x\le\frac{1}{2}\\ 2\pi+\cos^{-1}(4x^3-3x), & \text{if } -1\le x\le-\frac{1}{2}\end{cases}$
11.	• $2\tan^{-1}x=\begin{cases}\tan^{-1}\left(\frac{2x}{1-x^2}\right) & , \text{ if} -1<x<1\\ \pi+\tan^{-1}\left(\frac{2x}{1-x^2}\right) & , \text{ if } x>1\\ -\pi+\tan^{-1}\left(\frac{2x}{1-x^2}\right), & \text{if } x<-1\end{cases}$	• $3\tan^{-1}x=\begin{cases}\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right) & , \text{ if } -\frac{1}{\sqrt{3}}<x<\frac{1}{\sqrt{3}}\\ \pi+\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right) & , \text{ if } x>\frac{1}{\sqrt{3}}\\ -\pi+\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right), & \text{if } x<-\frac{1}{\sqrt{3}}\end{cases}$
12.	• $2\tan^{-1}x=\begin{cases}\sin^{-1}\left(\frac{2x}{1+x^2}\right) & , \text{ if } -1\le x\le 1\\ \pi-\sin^{-1}\left(\frac{2x}{1+x^2}\right) & , \text{ if } x>1\\ -\pi-\sin^{-1}\left(\frac{2x}{1+x^2}\right), & \text{if } x<-1\end{cases}$	• $2\tan^{-1}x=\begin{cases}\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) & , \text{ if } 0\le x<\infty\\ -\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), & \text{if } -\infty<x\le 0\end{cases}$
13.	• $\sin^{-1}x=\cos^{-1}\left(\sqrt{1-x^2}\right)=\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)=\cot^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)=\sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)=\text{cosec}^{-1}\left(\frac{1}{x}\right)$	
	• $\cos^{-1}x=\sin^{-1}\left(\sqrt{1-x^2}\right)=\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)=\cot^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)=\sec^{-1}\left(\frac{1}{x}\right)=\text{cosec}^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$	
	• $\tan^{-1}x=\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)=\cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)=\cot^{-1}\left(\frac{1}{x}\right)=\sec^{-1}\left(\sqrt{1+x^2}\right)=\text{cosec}^{-1}\left(\frac{\sqrt{1+x^2}}{x}\right)$	
14.	• If $x_1, x_2, \ldots, x_n \in R$, then $\tan^{-1}x_1+\tan^{-1}x_2+\ldots+\tan^{-1}x_n=\tan^{-1}\left(\frac{S_1-S_3+S_5-S_7+\ldots}{1-S_2+S_4-S_6+\ldots}\right)$ where S_k = Sum of the products of $x_1, x_2, \ldots, x_n$ taken k at a time.	

WORK IT OUT

VERY SHORT ANSWER TYPE

1. Find the value of $\cot\left(\frac{\pi}{4}-2\cot^{-1}3\right)$.

2. If $\tan^{-1}\frac{4}{3}=\theta$, find the value of $\cos\theta$.

3. Find the principal value of $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$.

4. For the principal values, evaluate the following:

$$\cot^{-1}(-\sqrt{3})+\tan^{-1}(1)+\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$$

5. Evaluate : $\cos^{-1}(\cos(-680°))$

SHORT ANSWER TYPE

6. Evaluate : **(i)** $\sin(\cot^{-1}x)$ **(ii)** $\cos(\tan^{-1}x)$

7. Prove that $\sin^{-1}\frac{8}{17}+\sin^{-1}\frac{3}{5}=\cos^{-1}\frac{36}{85}$.

8. Solve for x : $\cos(\tan^{-1}x)=\sin\left(\sec^{-1}\frac{13}{12}\right)$.

9. Solve : $\sin^{-1}x=\frac{\pi}{6}+\cos^{-1}x$

10. If in a ΔABC, $\angle A=\tan^{-1}2$ and $\angle B=\tan^{-1}3$, then show that $\angle C$ is equal to $\frac{\pi}{4}$.

LONG ANSWER TYPE-I

11. Write each of the following in the simplest form:

(i) $\tan^{-1}(\sec x+\tan x)$ **(ii)** $\sin\left(2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right)$

12. Show that $2\tan^{-1}\left(\frac{1+x}{1-x}\right)+\sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)=\pi$

13. Evaluate the following :

(i) $\sin(2\sin^{-1}0.8)$ **(ii)** $\tan\left[2\tan^{-1}\left(\frac{1}{5}\right)-\frac{\pi}{4}\right]$

14. Find the value of

$$\tan^{-1}\left(\tan\frac{5\pi}{6}\right)+\cos^{-1}\left(\cos\frac{13\pi}{6}\right)+\sec^{-1}\left(\sec\frac{9\pi}{5}\right)$$

15. If $\tan^{-1}\left\{\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right\}=\alpha$, then prove that $x^2=\sin 2\alpha$.

LONG ANSWER TYPE-II

16. If $a, b, c>0$ such that $a+b+c=abc$, find the value of $\tan^{-1}a+\tan^{-1}b+\tan^{-1}c$.

17. If $\cos^{-1}\frac{x}{a}+\cos^{-1}\frac{y}{b}=\alpha$, prove that

$$\frac{x^2}{a^2}-\frac{2xy}{ab}\cos\alpha+\frac{y^2}{b^2}=\sin^2\alpha.$$

18. If $a_1, a_2, a_3, ..., a_n$ is an arithmetic progression with common difference d, then show that

$$\tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{d}{1+a_2a_3}\right)+\tan^{-1}\left(\frac{d}{1+a_3a_4}\right)+...+\left(\frac{d}{1+a_{n-1}a_n}\right)\right]$$

is equal to $\frac{a_n-a_1}{1+a_1a_n}$.

19. Solve :

(i) $\cos\left(\sin^{-1}\frac{1}{3}+\cos^{-1}x\right)=0$

(ii) $\tan^{-1}\left(\frac{x+1}{x-1}\right)+\tan^{-1}\left(\frac{x-1}{x}\right)=\pi+\tan^{-1}(-7)$

20. Prove that

(i) $\sin^{-1}\frac{12}{13}+\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{63}{16}=\pi$

(ii) $\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{3}{5}=\tan^{-1}\frac{27}{11}$

SOLUTIONS

1. $\cot\left(\frac{\pi}{4}-2\cot^{-1}3\right)=\cot\left[\frac{\pi}{4}-\cot^{-1}\left(\frac{3^2-1}{2\times 3}\right)\right]$

$$=\cot\left(\frac{\pi}{4}-\cot^{-1}\frac{4}{3}\right)=\frac{\cot\frac{\pi}{4}\cdot\frac{4}{3}+1}{\frac{4}{3}-\cot\frac{\pi}{4}}=\frac{\frac{4}{3}+1}{\frac{4}{3}-1}=7$$

2. Given, $\tan^{-1}\frac{4}{3}=\theta$, where $\theta\in\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$

$\therefore\ \tan\theta=\frac{4}{3}$.

We know that $\cos\theta>0$, when $\theta\in\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$.

$$\therefore\ \cos\theta=\frac{1}{\sec\theta}=\frac{1}{\sqrt{1+\tan^2\theta}}=\frac{1}{\sqrt{1+\frac{16}{9}}}=\frac{3}{5}$$

3. Since, $\sin\frac{2\pi}{3}=\sin\left(\pi-\frac{\pi}{3}\right)=\sin\frac{\pi}{3}$

$\therefore\quad \sin^{-1}\left(\sin\frac{2\pi}{3}\right)=\sin^{-1}\left(\sin\frac{\pi}{3}\right)=\frac{\pi}{3}$.

Hence, the principal value of $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$ is $\frac{\pi}{3}$.

4. $\cot^{-1}(-\sqrt{3})+\tan^{-1}(1)+\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$

$=\cot^{-1}\cot\left(\frac{5\pi}{6}\right)+\tan^{-1}\left(\tan\frac{\pi}{4}\right)+\sec^{-1}\left(\sec\frac{\pi}{6}\right)$

$=\frac{5\pi}{6}+\frac{\pi}{4}+\frac{\pi}{6}=\frac{5\pi}{4}$

5. $\cos^{-1}\{\cos(-680°)\}=\cos^{-1}(\cos 680°)$

$=\cos^{-1}\left(\cos\frac{34\pi}{9}\right)=\cos^{-1}\left\{\cos\left(4\pi-\frac{2\pi}{9}\right)\right\}$

$=\cos^{-1}\left(\cos\frac{2\pi}{9}\right)=\frac{2\pi}{9}=40°$

6. **(i)** We have, $\sin(\cot^{-1}x)=\sin\left(\cot^{-1}\frac{x}{1}\right)$.

Now, $\cot^{-1}x=\sin^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$

Hence, $\sin(\cot^{-1}x)=\sin\left(\sin^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)\right)=\frac{1}{\sqrt{1+x^2}}$

(ii) $\cos(\tan^{-1}x)=\cos\left(\tan^{-1}\frac{x}{1}\right)$

Now, $\tan^{-1}x=\cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$

Hence, $\cos(\tan^{-1}x)=\cos\left(\cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)\right)=\frac{1}{\sqrt{1+x^2}}$

7. L.H.S. $=\sin^{-1}\frac{8}{17}+\sin^{-1}\frac{3}{5}=\cos^{-1}\frac{15}{17}+\cos^{-1}\frac{4}{5}$

$=\cos^{-1}\left\{\left(\frac{15}{17}\times\frac{4}{5}\right)-\sqrt{1-\left(\frac{15}{17}\right)^2}\cdot\sqrt{1-\left(\frac{4}{5}\right)^2}\right\}$

$=\cos^{-1}\left\{\frac{12}{17}-\sqrt{\frac{64}{289}}\cdot\sqrt{\frac{9}{25}}\right\}=\cos^{-1}\left\{\frac{12}{17}-\frac{8}{17}\times\frac{3}{5}\right\}$

$=\cos^{-1}\left\{\frac{12}{17}-\frac{24}{85}\right\}=\cos^{-1}\left(\frac{36}{85}\right)=$ R.H.S.

Hence, $\sin^{-1}\frac{8}{17}+\sin^{-1}\frac{3}{5}=\cos^{-1}\frac{36}{85}$.

8. Let $\tan^{-1}x=\phi$. Then, $\tan\phi=\frac{x}{1}$

$\therefore\quad \cos\phi=\frac{1}{\sqrt{1+x^2}}\Rightarrow\phi=\cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$

So, $\cos(\tan^{-1}x)=\cos\left(\cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)\right)=\frac{1}{\sqrt{1+x^2}}$

Also, let $\sec^{-1}\frac{13}{12}=\theta$. Then, $\sec\theta=\frac{13}{12}$

$\therefore\quad \sin\theta=\frac{5}{13}\Rightarrow\theta=\sin^{-1}\frac{5}{13}$

So, $\sin\left(\sec^{-1}\frac{13}{12}\right)=\sin\left(\sin^{-1}\frac{5}{13}\right)=\frac{5}{13}$

Thus, $\frac{1}{\sqrt{1+x^2}}=\frac{5}{13}\quad\Rightarrow\quad\frac{1}{(1+x^2)}=\frac{25}{169}$

$\Rightarrow\ 1+x^2=\frac{169}{25}\Rightarrow x^2=\frac{169}{25}-1\Rightarrow x=\pm\frac{144}{25}$

Hence, $x=\pm\frac{12}{5}$.

9. We have, $\sin^{-1}x=\frac{\pi}{6}+\cos^{-1}x$

$\Rightarrow\ \sin^{-1}x-\cos^{-1}x=\frac{\pi}{6}$

$\Rightarrow\ \sin^{-1}x-\left(\frac{\pi}{2}-\sin^{-1}x\right)=\frac{\pi}{6}$

$\Rightarrow\ 2\sin^{-1}x=\frac{2\pi}{3}\ \Rightarrow\ \sin^{-1}x=\frac{\pi}{3}\ \Rightarrow\ x=\frac{\sqrt{3}}{2}$

10. We have, $\angle A=\tan^{-1}2$, $\angle B=\tan^{-1}3$

We know that, $\angle A+\angle B+\angle C=\pi$

$\Rightarrow\ \tan^{-1}2+\tan^{-1}3+\angle C=\pi$

$\Rightarrow\ \pi+\tan^{-1}\left(\frac{2+3}{1-2\times3}\right)+\angle C=\pi$

$\Rightarrow\ \pi+\tan^{-1}(-1)+\angle C=\pi\Rightarrow\pi-\frac{\pi}{4}+\angle C=\pi$

$\Rightarrow\ \frac{3\pi}{4}+\angle C=\pi\ \Rightarrow\ \angle C=\frac{\pi}{4}$

11. **(i)** $\sec x+\tan x=\frac{1+\sin x}{\cos x}=\frac{1-\cos\left(\frac{\pi}{2}+x\right)}{\sin\left(\frac{\pi}{2}+x\right)}$

$=\frac{2\sin^2\left(\frac{\pi}{4}+\frac{x}{2}\right)}{2\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)}=\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)$

$$\therefore \tan^{-1}\left(\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right)=\frac{\pi}{4}+\frac{x}{2}$$

(ii) Put $x = \cos\theta$

$$\therefore \sin\left(2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right)=\sin\left(2\tan^{-1}\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}\right)$$

$$=\sin\left(2\tan^{-1}\sqrt{\frac{2\sin^2\theta/2}{2\cos^2\theta/2}}\right)$$

$$=\sin\left[2\tan^{-1}\left(\tan\frac{\theta}{2}\right)\right]=\sin\theta=\sqrt{1-x^2}$$

12. Given expression is $2\tan^{-1}\left(\frac{1+x}{1-x}\right)+\sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

Putting $x = \tan\theta$, we get

$$2\tan^{-1}\left(\frac{1+\tan\theta}{1-\tan\theta}\right)+\sin^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$$

$$=2\tan^{-1}\left(\frac{\tan\frac{\pi}{4}+\tan\theta}{1-\tan\frac{\pi}{4}\cdot\tan\theta}\right)+\sin^{-1}(\cos 2\theta)$$

$$=2\tan^{-1}\left[\tan\left(\frac{\pi}{4}+\theta\right)\right]+\sin^{-1}\left[\sin\left(\frac{\pi}{2}-2\theta\right)\right]$$

$$=2\left(\frac{\pi}{4}+\theta\right)+\frac{\pi}{2}-2\theta=\frac{\pi}{2}+2\theta+\frac{\pi}{2}-2\theta=\pi$$

13. **(i)** $\sin(2\sin^{-1}0.8)=\sin\left[\sin^{-1}\left(2\left(0.8\sqrt{1-(0.8)^2}\right)\right)\right]$

$= \sin[\sin^{-1}(2 \times 0.8 \times 0.6)]$

$= \sin[\sin^{-1}(0.96)] = 0.96$

(ii) $\tan\left[2\tan^{-1}\frac{1}{5}-\frac{\pi}{4}\right]=\tan\left[\tan^{-1}\left(\frac{2\left(\frac{1}{5}\right)}{1-\left(\frac{1}{5}\right)^2}\right)-\frac{\pi}{4}\right]$

$$=\tan\left[\tan^{-1}\frac{5}{12}-\frac{\pi}{4}\right]=\tan\left[\tan^{-1}\frac{5}{12}-\tan^{-1}1\right]$$

$$=\tan\left[\tan^{-1}\left(\frac{\frac{5}{12}-1}{1+\frac{5}{12}\times 1}\right)\right]=\tan\left[\tan^{-1}\left(\frac{-7}{17}\right)\right]=-\frac{7}{17}$$

14. $\tan^{-1}\left(\tan\frac{5\pi}{6}\right)+\cos^{-1}\left(\cos\frac{13\pi}{6}\right)+\sec^{-1}\left(\sec\frac{9\pi}{5}\right)$

$$\Rightarrow \tan^{-1}\left[\tan\left(\pi-\frac{\pi}{6}\right)\right]+\cos^{-1}\left[\cos\left(2\pi+\frac{\pi}{6}\right)\right]+\sec^{-1}\left[\sec\left(2\pi-\frac{\pi}{5}\right)\right]$$

$$\Rightarrow \tan^{-1}\left(-\tan\left(\frac{\pi}{6}\right)\right)+\cos^{-1}\left[\cos\left(\frac{\pi}{6}\right)\right]+\sec^{-1}\left[\sec\left(\frac{\pi}{5}\right)\right]$$

$$=-\frac{\pi}{6}+\frac{\pi}{6}+\frac{\pi}{5}=\frac{\pi}{5}.$$

15. We have, $\tan^{-1}\left\{\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right\}=\alpha$

$$\Rightarrow \frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}=\tan\alpha$$

$$\Rightarrow \frac{\left(\sqrt{1+x^2}-\sqrt{1-x^2}\right)+\left(\sqrt{1+x^2}+\sqrt{1-x^2}\right)}{\left(\sqrt{1+x^2}-\sqrt{1-x^2}\right)-\left(\sqrt{1+x^2}+\sqrt{1-x^2}\right)}=\frac{\tan\alpha+1}{\tan\alpha-1}$$

$$\Rightarrow \frac{2\sqrt{1+x^2}}{-2\sqrt{1-x^2}}=\frac{\tan\alpha+1}{\tan\alpha-1}\Rightarrow\frac{\sqrt{1-x^2}}{\sqrt{1+x^2}}=\frac{1-\tan\alpha}{1+\tan\alpha}$$

$$\Rightarrow \sqrt{\frac{1-x^2}{1+x^2}}=\frac{\cos\alpha-\sin\alpha}{\cos\alpha+\sin\alpha}$$

$$\Rightarrow \frac{1-x^2}{1+x^2}=\frac{1-\sin 2\alpha}{1+\sin 2\alpha}\Rightarrow x^2=\sin 2\alpha$$

16. It is given that $a + b + c = abc$

$$\therefore \frac{abc}{c}=\frac{a}{c}+\frac{b}{c}+1$$

$$\Rightarrow ab=1+\left(\frac{a}{c}+\frac{b}{c}\right)\quad\Rightarrow\quad ab-1=\frac{a+b}{c}$$

$\Rightarrow ab - 1 > 0 \qquad \left[\because\ a, b, c > 0\ \therefore \frac{a+b}{c}>0\right]$

$\Rightarrow ab > 1$

Now, $\tan^{-1} a + \tan^{-1} b + \tan^{-1} c$

$$=\pi+\tan^{-1}\left(\frac{a+b}{1-ab}\right)+\tan^{-1}c \qquad [\because\ ab > 1]$$

$$=\pi+\tan^{-1}\left(\frac{abc-c}{1-ab}\right)+\tan^{-1}c$$

$$=\pi+\tan^{-1}\left(\frac{-c(1-ab)}{1-ab}\right)+\tan^{-1}c$$

$= \pi + \tan^{-1}(-c) + \tan^{-1} c = \pi - \tan^{-1} c + \tan^{-1} c = \pi$

17. We have $\cos^{-1}\frac{x}{a}+\cos^{-1}\frac{y}{b}=\alpha$

$$\Rightarrow \cos^{-1}\left[\frac{x}{a}\cdot\frac{y}{b}-\sqrt{\left(1-\frac{x^2}{a^2}\right)\left(1-\frac{y^2}{b^2}\right)}\right]=\alpha$$

$$\Rightarrow \frac{xy}{ab}-\sqrt{1-\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{x^2y^2}{a^2b^2}}=\cos\alpha$$

$$\Rightarrow \frac{xy}{ab}-\cos\alpha=\sqrt{1-\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{x^2y^2}{a^2b^2}}$$

Squaring both sides, we have

$$\frac{x^2y^2}{a^2b^2}-\frac{2xy}{ab}\cos\alpha+\cos^2\alpha=1-\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{x^2y^2}{a^2b^2}$$

$$\Rightarrow \frac{x^2}{a^2}-\frac{2xy}{ab}\cos\alpha+\frac{y^2}{b^2}=1-\cos^2\alpha$$

$$\therefore \frac{x^2}{a^2}-\frac{2xy}{ab}\cos\alpha+\frac{y^2}{b^2}=\sin^2\alpha$$

18. Given, $a_1, a_2, a_3, a_4, \ldots, a_n$ is an arithmetic progression, then $d=a_2-a_1=a_3-a_2=\ldots=a_n-a_{n-1}$

$$\therefore \tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{d}{1+a_2a_3}\right)\right.$$
$$\left.+\tan^{-1}\left(\frac{d}{1+a_3a_4}\right)+\ldots+\tan^{-1}\left(\frac{d}{1+a_{n-1}a_n}\right)\right]$$

$$=\tan\left[\tan^{-1}\left(\frac{a_2-a_1}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{a_3-a_2}{1+a_2a_3}\right)\right.$$
$$\left.+\tan^{-1}\left(\frac{a_4-a_3}{1+a_3a_4}\right)+\ldots+\tan^{-1}\left(\frac{a_n-a_{n-1}}{1+a_{n-1}a_n}\right)\right]$$

$= \tan[\tan^{-1}a_2-\tan^{-1}a_1+\tan^{-1}a_3-\tan^{-1}a_2+\ldots+\tan^{-1}a_n-\tan^{-1}a_{n-1}]$

$=\tan[\tan^{-1}a_n-\tan^{-1}a_1]$

$$=\tan\left[\tan^{-1}\left(\frac{a_n-a_1}{1+a_na_1}\right)\right]=\frac{a_n-a_1}{1+a_1a_n}.$$

19. **(i)** Here, $\cos\left(\sin^{-1}\frac{1}{3}+\cos^{-1}x\right)=0=\cos\left(\pm\frac{\pi}{2}\right)$

$$\therefore \sin^{-1}\frac{1}{3}+\cos^{-1}x=\pm\frac{\pi}{2}$$

i.e., $\cos^{-1}x=\pm\frac{\pi}{2}-\sin^{-1}\frac{1}{3}$

$$\therefore x=\cos\left(\pm\frac{\pi}{2}-\sin^{-1}\frac{1}{3}\right)$$

$$x=\cos\left(\frac{\pi}{2}-\sin^{-1}\frac{1}{3}\right)=\sin\left(\sin^{-1}\frac{1}{3}\right)=\frac{1}{3}$$

$$x=\cos\left(-\frac{\pi}{2}-\sin^{-1}\frac{1}{3}\right)=\cos\left(\frac{\pi}{2}+\sin^{-1}\frac{1}{3}\right)=-\sin\left(\sin^{-1}\frac{1}{3}\right)=-\frac{1}{3}$$

$$\therefore x=\pm\frac{1}{3}$$

(ii) We have,

$$\tan^{-1}\left(\frac{x+1}{x-1}\right)+\tan^{-1}\left(\frac{x-1}{x}\right)=\pi+\tan^{-1}(-7)$$

$$\Rightarrow \tan^{-1}\left(\frac{\frac{x+1}{x-1}+\frac{x-1}{x}}{1-\frac{x+1}{x-1}\frac{x-1}{x}}\right)=\pi+\tan^{-1}(-7)$$

$$\therefore \frac{(x+1)x+(x-1)^2}{(x-1)x-(x+1)(x-1)}=\tan\left[\pi+\tan^{-1}(-7)\right]$$

$\Rightarrow x^2+x+x^2-2x+1=-7(x^2-x-x^2+1)$

$\Rightarrow 2x^2-8x+8=0$ *i.e.,* $x^2-4x+4=0$

$\Rightarrow (x-2)^2=0$

$\therefore x=2$

20. **(i)** L.H.S. $=\sin^{-1}\frac{12}{13}+\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{63}{16}$

$$=\tan^{-1}\frac{12}{5}+\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{63}{16}$$

$$=\pi+\tan^{-1}\left(\frac{\frac{12}{5}+\frac{3}{4}}{1-\frac{12}{5}\times\frac{3}{4}}\right)+\tan^{-1}\frac{63}{16}$$

$$=\pi+\tan^{-1}\left(-\frac{63}{16}\right)+\tan^{-1}\frac{63}{16}$$

$$=\pi-\tan^{-1}\frac{63}{16}+\tan^{-1}\frac{63}{16}=\pi$$

(ii) Let $\cos^{-1}\frac{4}{5}=\theta$. Then $\cos\theta=\frac{4}{5}$.

$$\therefore \tan\theta=\frac{\sqrt{1-\cos^2\theta}}{\cos\theta}=\frac{3}{4}\Rightarrow\theta=\tan^{-1}\frac{3}{4}$$

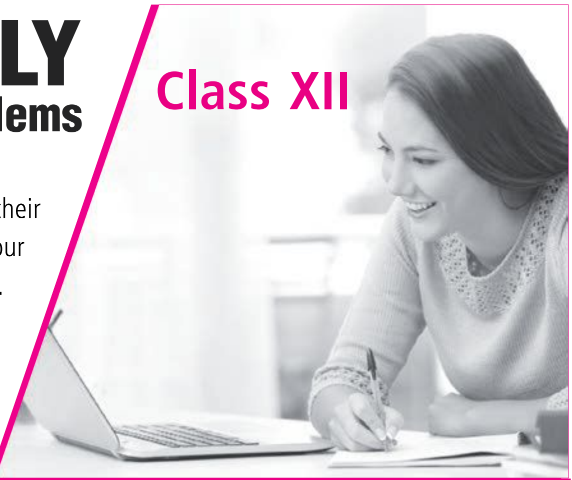
Consequently, $\cos^{-1}\frac{4}{5}=\tan^{-1}\frac{3}{4}$

$$\therefore \cos^{-1}\frac{4}{5}+\tan^{-1}\frac{3}{5}=\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{3}{5}$$

$$=\tan^{-1}\left(\frac{\frac{3}{4}+\frac{3}{5}}{1-\frac{3}{4}\cdot\frac{3}{5}}\right)=\tan^{-1}\frac{27}{11}$$

MPP-2 MONTHLY Practice Problems

Class XII



This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Inverse Trigonometric Functions

Total Marks : 80 **Time Taken : 60 Min.**

Only One Option Correct Type

1. The number of positive integral solutions of

$$\tan^{-1}x+\cos^{-1}\frac{y}{\sqrt{1+y^2}}=\sin^{-1}\frac{3}{\sqrt{10}} \text{ is}$$

(a) 0 (b) 1 (c) 2 (d) > 2

2. If $\left|\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right|<\frac{\pi}{3}$, then x belongs to the interval

(a) $\left[-\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right]$ (b) $\left(-\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right)$

(c) $\left(0,\frac{1}{\sqrt{3}}\right)$ (d) none of these

3. $\cos^{-1}\left[\cos\left(-\frac{17\pi}{15}\right)\right]$ is equal to

(a) $\frac{17\pi}{15}$ (b) $\frac{13\pi}{15}$

(c) $\frac{3\pi}{15}$ (d) $-\frac{17\pi}{15}$

4. The domain of the function

$$f(x)=\cos^{-1}\left(\frac{2-|x|}{4}\right)+[\log(3-x)]^{-1} \text{ is}$$

(a) $[-6, 6]$ (b) $[-5, 2)\cup(2, 3)$

(c) $(2, 3)$ (d) $[-6, 2)\cup(2, 3)$

5. If θ are ϕ are the roots of the equation $8x^2 + 22x + 5 = 0$, then

(a) both $\sin^{-1}\theta$ and $\sin^{-1}\phi$ are real

(b) both $\sec^{-1}\theta$ and $\sec^{-1}\phi$ are real

(c) both $\tan^{-1}\theta$ and $\tan^{-1}\phi$ are real

(d) none of these

6. If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{3\pi}{2}$, then the value of $25\,(x+y+z) - \frac{216}{(x^3+y^3+z^3)}$ must be

(a) 1 (b) 2

(c) 3 (d) none of these

One or More Than One Option(s) Correct Type

7. If $\cos^{-1}x + (\sin^{-1}y)^2 = \frac{p\pi^2}{4}$ and

$(\cos^{-1}x)(\sin^{-1}y)^2 = \frac{\pi^2}{16}$, then

(a) $0\le p\le\frac{4}{\pi}+1$

(b) $p = 2$ is the integral value of p

(c) $p = 0, 1, 2$ (integral values)

(d) none of these

8. For $0<\phi<\frac{\pi}{2}$, if $x=\sum_{n=0}^{\infty}\cos^{2n}\phi$, $y=\sum_{n=0}^{\infty}\sin^{2n}\phi$, $z=\sum_{n=0}^{\infty}\cos^{2n}\phi\sin^{2n}\phi$, then

(a) $xyz = xz + y$ (b) $xyz = xy + z$

(c) $xyz = x + y + z$ (d) $xyz = yz + x$

9. For the equation

$2x = \tan(2\tan^{-1}a) + 2\tan(\tan^{-1}a + \tan^{-1}a^3)$,

which of the following is invalid?

(a) $a^2x + 2a = x$ (b) $a^2 + 2ax + 1 = 0$

(c) $a \ne 0$ (d) $a \ne -1, 1$

10. $\tan^{-1}\left(\frac{a_1x-y}{a_1y+x}\right)+\tan^{-1}\left(\frac{a_2-a_1}{a_1a_2+1}\right)+\tan^{-1}\left(\frac{a_3-a_2}{a_2a_3+1}\right)$ $+\ldots\ldots+\tan^{-1}\left(\frac{a_n-a_{n-1}}{a_{n-1}a_n+1}\right)+\tan^{-1}\frac{1}{a_n}$ is equal to

(a) $\tan^{-1} xy$ (b) $\tan^{-1}\frac{x}{y}$

(c) $\tan^{-1}\frac{y}{x}$ (d) none of these

11. The value of θ for which

$$\theta = \tan^{-1}(2\tan^2\theta) - \frac{1}{2}\sin^{-1}\left(\frac{3\sin 2\theta}{5+4\cos 2\theta}\right), \text{ is/are:}$$

(a) $n\pi + \tan^{-1}(-2)$ (b) $n\pi,\ n\pi + \frac{\pi}{4}$

(c) $n\pi + \cot^{-1}(-2)$ (d) none of these

12. If x, y and z are in A.P. and $\tan^{-1} x$, $\tan^{-1} y$ and $\tan^{-1} z$ are also in A.P. then

(a) $x = y = z$ (b) $2x = 3y = 6z$

(c) $6x = 3y = 2z$ (d) $6x = 4y = 3z$

13. In ΔABC, $\angle C = \frac{\pi}{2}$ and

$$\sin^{-1} x = \sin^{-1}\left(\frac{ax}{c}\right) + \sin^{-1}\left(\frac{bx}{c}\right)$$

where a, b and c are the sides of triangle, then the values of x is/are

(a) 0 (b) 1 (c) 2 (d) −1

Comprehension Type

$$\sum_{r=1}^{n}\tan^{-1}\left(\frac{x_r - x_{r-1}}{1+x_{r-1}x_r}\right) = \sum_{r=1}^{n}\left(\tan^{-1}x_r - \tan^{-1}x_{r-1}\right)$$
$$= \tan^{-1}x_n - \tan^{-1}x_0, \forall n \in N$$

On the basis of above information, answer the following questions:

14. The value of $\operatorname{cosec}^{-1}\sqrt{5} + \operatorname{cosec}^{-1}\sqrt{65} + \operatorname{cosec}^{-1}\sqrt{(325)} + \ldots$ to ∞ is

(a) π (b) $\frac{3\pi}{4}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

15. The sum to infinite terms of the series

$$\tan^{-1}\left(\frac{2}{1-1^2+1^4}\right) + \tan^{-1}\left(\frac{4}{1-2^2+2^4}\right) + \tan^{-1}\left(\frac{6}{1-3^2+3^4}\right) + \ldots \text{ is}$$

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$

(c) $\frac{3\pi}{4}$ (d) none of these

Matrix Match Type

16. Match the following :

	Column I		Column II
P.	If $2\tan^{-1}(2x+1) = \cos^{-1}(-x)$, then x is	1.	$-\frac{1}{\sqrt{2}}$
Q.	If $2\cos^{-1}x = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$, then x is	2.	$\frac{\sqrt{3}}{2}$
R.	If $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$, then x is	3.	−1
		4.	0

	P	Q	R
(a)	2	1	3
(b)	3	2	1
(c)	3	1	2
(d)	4	2	1

Integer Answer Type

17. The number of solutions for the equation

$$2\sin^{-1}\sqrt{(x^2-x+1)} + \cos^{-1}\sqrt{(x^2-x)} = \frac{3\pi}{2} \text{ is}$$

18. If $\tan\theta + \tan\left(\frac{\pi}{3}+\theta\right) + \tan\left(-\frac{\pi}{3}+\theta\right) = a\tan 3\theta$, then a is equal to

19. If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{3\pi}{2}$ and $f(1) = 2$, $f(p+q) = f(p).f(q)$, $\forall\ p, q \in R$, then $x^{f(1)} + y^{f(2)} + z^{f(3)} - \frac{(x+y+z)}{x^{f(1)}+y^{f(2)}+z^{f(3)}}$ is equal to

20. If $\lambda = \tan\left(2\tan^{-1}\frac{1}{5} - \frac{\pi}{4}\right)$, then the value of $-17\lambda^2$ must be

◈◈

Keys are published in this issue. Search now! ☺

SELF CHECK

No. of questions attempted ……

No. of questions correct ……

Marks scored in percentage ……

Check your score! If your score is

> 90%	EXCELLENT WORK !	You are well prepared to take the challenge of final exam.
90-75%	GOOD WORK !	You can score good in the final exam.
74-60%	SATISFACTORY !	You need to score more next time.
< 60%	NOT SATISFACTORY!	Revise thoroughly and strengthen your concepts.

WB JEE
SOLVED PAPER 2017

CATEGORY-I (Q. 1 to Q. 50)

Only one answer is correct. Correct answer will fetch full marks 1. Incorrect answer or any combination of more than one answer will fetch –¼ marks. No answer will fetch 0 marks.

1. The number of all numbers having 5 digits, with distinct digits is
   (a) 99999 (b) $9 \times {}^9P_4$ (c) ${}^{10}P_5$ (d) ${}^9P_4$

2. The greatest integer which divides $(p+1)(p+2)(p+3).....(p+q)$ for all $p \in N$ and fixed $q \in N$ is
   (a) $p!$ (b) $q!$ (c) p (d) q

3. Let $(1 + x + x^2)^9 = a_0 + a_1x + a_2x^2 + ... + a_{18}x^{18}$. Then
   (a) $a_0 + a_2 + ... + a_{18} = a_1 + a_3 + ... + a_{17}$
   (b) $a_0 + a_2 + ... + a_{18}$ is even
   (c) $a_0 + a_2 + ... + a_{18}$ is divisible by 9
   (d) $a_0 + a_2 + ... + a_{18}$ is divisible by 3 but not by 9

4. The linear system of equations $\left.\begin{array}{l} 8x-3y-5z=0 \\ 5x-8y+3z=0 \\ 3x+5y-8z=0 \end{array}\right\}$ has
   (a) only zero solution
   (b) only finite number of non-zero solutions
   (c) no non-zero solution
   (d) infinitely many non-zero solutions

5. Let P be the set of all non-singular matrices of order 3 over R and Q be the set of all orthogonal matrices of order 3 over R. Then
   (a) P is proper subset of Q
   (b) Q is proper subset of P
   (c) Neither P is proper subset of Q nor Q is proper subset of P
   (d) $P \cap Q = \phi$, the void set

6. Let $A = \begin{pmatrix} x+2 & 3x \\ 3 & x+2 \end{pmatrix}, B = \begin{pmatrix} x & 0 \\ 5 & x+2 \end{pmatrix}$. Then all solutions of the equation det $(AB) = 0$ is
   (a) 1, –1, 0, 2 (b) 1, 4, 0, –2
   (c) 1, –1, 4, 3 (d) –1, 4, 0, 3

7. The value of det A, where
   $A = \begin{pmatrix} 1 & \cos\theta & 0 \\ -\cos\theta & 1 & \cos\theta \\ -1 & -\cos\theta & 1 \end{pmatrix}$ lies
   (a) in the closed interval [1, 2]
   (b) in the closed interval [0, 1]
   (c) in the open interval (0, 1)
   (d) in the open interval (1, 2)

8. Let $f : R \to R$ be such that f is injective and $f(x)f(y) = f(x+y)$, $\forall x, y \in R$. If $f(x), f(y), f(z)$ are in G.P., then x, y, z are in
   (a) A.P. always
   (b) G.P. always
   (c) A.P. depending on the value of x, y, z
   (d) G.P. depending on the value of x, y, z

9. On the set R of real numbers we define xPy if and only if $xy \geq 0$. Then the relation P is
   (a) reflexive but not symmetric
   (b) symmetric but not reflexive
   (c) transitive but not reflexive
   (d) reflexive and symmetric but not transitive

10. On R, the relation ρ be defined by '$x\rho y$ holds if and only if $x - y$ is zero or irrational'. Then
    (a) ρ is reflexive and transitive but not symmetric.
    (b) ρ is reflexive and symmetric but not transitive.
    (c) ρ is symmetric and transitive but not reflexive.
    (d) ρ is equivalence relation

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11. Mean of n observations x_1, x_2, ..., x_n is $\bar{x}$. If an observation x_q is replaced by x'_q then the new mean is

(a) $\bar{x} - x_q + x'_q$ (b) $\dfrac{(n-1)\bar{x} + x'_q}{n}$

(c) $\dfrac{(n-1)\bar{x} - x'_q}{n}$ (d) $\dfrac{n\bar{x} - x_q + x'_q}{n}$

12. The probability that a non leap year selected at random will have 53 Sundays is

(a) 0 (b) 1/7 (c) 2/7 (d) 3/7

13. The equation $\sin x(\sin x + \cos x) = k$ has real solutions, where k is a real number. Then

(a) $0 \le k \le \dfrac{1+\sqrt{2}}{2}$ (b) $2-\sqrt{3} \le k \le 2+\sqrt{3}$

(c) $0 \le k \le 2-\sqrt{3}$ (d) $\dfrac{1-\sqrt{2}}{2} \le k \le \dfrac{1+\sqrt{2}}{2}$

14. The possible values of x, which satisfy the trigonometric equation

$\tan^{-1}\left(\dfrac{x-1}{x-2}\right) + \tan^{-1}\left(\dfrac{x+1}{x+2}\right) = \dfrac{\pi}{4}$ are

(a) $\pm\dfrac{1}{\sqrt{2}}$ (b) $\pm\sqrt{2}$ (c) $\pm\dfrac{1}{2}$ (d) ± 2

15. Transforming to parallel axes through a point (p, q), the equation $2x^2 + 3xy + 4y^2 + x + 18y + 25 = 0$ becomes $2x^2 + 3xy + 4y^2 = 1$. Then

(a) $p = -2, q = 3$ (b) $p = 2, q = -3$

(c) $p = 3, q = -4$ (d) $p = -4, q = 3$

16. Let $A(2, -3)$ and $B(-2, 1)$ be two angular points of ΔABC. If the centroid of the triangle moves on the line $2x + 3y = 1$, then the locus of the angular point C is given by

(a) $2x + 3y = 9$ (b) $2x - 3y = 9$

(c) $3x + 2y = 5$ (d) $3x - 2y = 3$

17. The point $P(3, 6)$ is first reflected on the line $y = x$ and then the image point Q is again reflected on the line $y = -x$ to get the image point Q'. Then the circumcentre of the $\Delta PQQ'$ is

(a) (6, 3) (b) (6, –3) (c) (3, –6) (d) (0, 0)

18. Let d_1 and d_2 be the lengths of the perpendiculars drawn from any point of the line $7x - 9y + 10 = 0$ upon the lines $3x + 4y = 5$ and $12x + 5y = 7$ respectively. Then

(a) $d_1 > d_2$ (b) $d_1 = d_2$ (c) $d_1 < d_2$ (d) $d_1 = 2d_2$

19. The common chord of the circles $x^2 + y^2 - 4x - 4y = 0$ and $2x^2 + 2y^2 = 32$ subtends at the origin an angle equal to

(a) $\dfrac{\pi}{3}$ (b) $\dfrac{\pi}{4}$ (c) $\dfrac{\pi}{6}$ (d) $\dfrac{\pi}{2}$

20. The locus of the mid-points of the chords of the circle $x^2 + y^2 + 2x - 2y - 2 = 0$ which make an angle of 90° at the centre is

(a) $x^2 + y^2 - 2x - 2y = 0$

(b) $x^2 + y^2 - 2x + 2y = 0$

(c) $x^2 + y^2 + 2x - 2y = 0$

(d) $x^2 + y^2 + 2x - 2y - 1 = 0$

21. Let P be the foot of the perpendicular from focus S of hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ on the line $bx - ay = 0$ and let C be the centre of the hyperbola. Then the area of the rectangle whose sides are equal to that of SP and CP is

(a) $2ab$ (b) ab

(c) $\dfrac{(a^2+b^2)}{2}$ (d) $\dfrac{a}{b}$

22. B is an extremity of the minor axis of an ellipse whose foci are S and S'. If $\angle SBS'$ is a right angle, then the eccentricity of the ellipse is

(a) $\dfrac{1}{2}$ (b) $\dfrac{1}{\sqrt{2}}$ (c) $\dfrac{2}{3}$ (d) $\dfrac{1}{3}$

23. The axis of the parabola $x^2 + 2xy + y^2 - 5x + 5y - 5 = 0$ is

(a) $x + y = 0$ (b) $x + y - 1 = 0$

(c) $x - y + 1 = 0$ (d) $x - y = \dfrac{1}{\sqrt{2}}$

24. The line segment joining the foci of the hyperbola $x^2 - y^2 + 1 = 0$ is one of the diameters of a circle. The equation of the circle is

(a) $x^2 + y^2 = 4$ (b) $x^2 + y^2 = \sqrt{2}$

(c) $x^2 + y^2 = 2$ (d) $x^2 + y^2 = 2\sqrt{2}$

25. The equation of the plane through (1, 2, –3) and (2, –2, 1) and parallel to X-axis is

(a) $y - z + 1 = 0$ (b) $y - z - 1 = 0$

(c) $y + z - 1 = 0$ (d) $y + z + 1 = 0$

26. Three lines are drawn from the origin O with direction cosines proportional to (1, –1, 1), (2, –3, 0) and (1, 0, 3). The three lines are

(a) not coplanar (b) coplanar

(c) perpendicular to each other
(d) coincident

27. Consider the non-constant differentiable function f of one variable which obeys the relation $\dfrac{f(x)}{f(y)} = f(x-y)$. If $f'(0) = p$ and $f'(5) = q$, then $f'(-5)$ is

(a) $\dfrac{p^2}{q}$ (b) $\dfrac{q}{p}$ (c) $\dfrac{p}{q}$ (d) q

28. If $f(x) = \log_5 \log_3 x$, then $f'(e)$ is equal to

(a) $e \log_e 5$ (b) $e \log_e 3$

(c) $\dfrac{1}{e \log_e 5}$ (d) $\dfrac{1}{e \log_e 3}$

29. Let $F(x) = e^x$, $G(x) = e^{-x}$ and $H(x) = G(F(x))$, where x is a real variable. Then $\dfrac{dH}{dx}$ at $x = 0$ is

(a) 1 (b) -1 (c) $-\dfrac{1}{e}$ (d) $-e$

30. If $f''(0) = k$, $k \neq 0$, then the value of

$\lim\limits_{x\to 0} \dfrac{2f(x)-3f(2x)+f(4x)}{x^2}$ is

(a) k (b) $2k$ (c) $3k$ (d) $4k$

31. If $y = e^{m\sin^{-1}x}$, then $(1 - x^2)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} - ky = 0$, where k is equal to

(a) m^2 (b) 2 (c) -1 (d) $-m^2$

32. The chord of the curve $y = x^2 + 2ax + b$, joining the points where $x = \alpha$ and $x = \beta$, is parallel to the tangent to the curve at abscissa $x =$

(a) $\dfrac{a+b}{2}$ (b) $\dfrac{2a+b}{3}$ (c) $\dfrac{2\alpha+\beta}{3}$ (d) $\dfrac{\alpha+\beta}{2}$

33. Let $f(x) = x^{13} + x^{11} + x^9 + x^7 + x^5 + x^3 + x + 19$. Then $f(x) = 0$ has

(a) 13 real roots
(b) only one positive and only two negative real roots
(c) not more than one real root
(d) has two positive and one negative real root

34. Let $f(x) = \begin{cases} \dfrac{x^p}{(\sin x)^q}, & \text{if } 0 < x \le \dfrac{\pi}{2} \\ 0, & \text{if } x = 0 \end{cases}$, $(p, q \in R)$. Then Lagrange's mean value theorem is applicable to $f(x)$ in closed interval $[0, x]$,

(a) for all p, q (b) only when $p > q$
(c) only when $p < q$ (d) for no value of p, q

35. $\lim\limits_{x\to 0} (\sin x)^{2\tan x}$

(a) is 2 (b) is 1
(c) is 0 (d) does not exist

36. $\int \cos(\log x)dx = F(x) + c$, where c is an arbitrary constant. Here $F(x) =$

(a) $x[\cos(\log x) + \sin(\log x)]$
(b) $x[\cos(\log x) - \sin(\log x)]$
(c) $\dfrac{x}{2}[\cos(\log x) + \sin(\log x)]$
(d) $\dfrac{x}{2}[\cos(\log x) - \sin(\log x)]$

37. $\dfrac{x^2-1}{x^4+3x^2+1}dx$ $(x > 0)$ is

(a) $\tan^{-1}\left(x+\dfrac{1}{x}\right)+c$ (b) $\tan^{-1}\left(x-\dfrac{1}{x}\right)+c$

(c) $\log_e\left|\dfrac{x+\frac{1}{x}-1}{x+\frac{1}{x}+1}\right|+c$ (d) $\log_e\left|\dfrac{x-\frac{1}{x}-1}{x-\frac{1}{x}+1}\right|+c$

38. Let $I = \displaystyle\int_{10}^{19} \frac{\sin x}{1+x^8}dx$. Then,

(a) $|I| < 10^{-9}$ (b) $|I| < 10^{-7}$
(c) $|I| < 10^{-5}$ (d) $|I| > 10^{-7}$

39. Let $I_1 = \displaystyle\int_0^n [x]dx$ and $I_2 = \displaystyle\int_0^n \{x\}dx$, where $[x]$ and $\{x\}$ are integral and fractional parts of x and $n \in N - \{1\}$. Then I_1/I_2 is equal to

(a) $\dfrac{1}{n-1}$ (b) $\dfrac{1}{n}$ (c) n (d) $n - 1$

40. The value of $\lim\limits_{n\to\infty}\left[\dfrac{n}{n^2+1^2}+\dfrac{n}{n^2+2^2}+.....+\dfrac{1}{2n}\right]$ is

(a) $\dfrac{n\pi}{4}$ (b) $\dfrac{\pi}{4}$ (c) $\dfrac{\pi}{4n}$ (d) $\dfrac{\pi}{2n}$

41. The value of the integral $\displaystyle\int_0^1 e^{x^2}dx$

(a) is less than 1
(b) is greater than 1
(c) is less than or equal to 1
(d) lies in the closed interval $[1, e]$

42. $\int_0^{100} e^{x-[x]}dx =$

(a) $\frac{e^{100}-1}{100}$ (b) $\frac{e^{100}-1}{e-1}$

(c) $100(e-1)$ (d) $\frac{e-1}{100}$

43. Solution of $(x+y)^2\frac{dy}{dx} = a^2$ ('a' being a constant) is

(a) $\frac{(x+y)}{a} = \tan\frac{y+c}{a}, c$ is an arbitrary constant

(b) $xy = a \tan cx$, c is an arbitrary constant

(c) $\frac{x}{a} = \tan\frac{y}{c}, c$ is an arbitrary constant

(d) $xy = \tan(x+c)$, c is an arbitrary constant

44. The integrating factor of the first order differential equation

$x^2(x^2-1)\frac{dy}{dx} + x(x^2+1)y = x^2 - 1$ is

(a) e^x (b) $x-\frac{1}{x}$ (c) $x+\frac{1}{x}$ (d) $\frac{1}{x^2}$

45. In a G.P. series consisting of positive terms, each term is equal to the sum of next two terms. Then the common ratio of this G.P. series is

(a) $\sqrt{5}$ (b) $\frac{\sqrt{5}-1}{2}$ (c) $\frac{\sqrt{5}}{2}$ (d) $\frac{\sqrt{5}+1}{2}$

46. If $(\log_5 x)(\log_x 3x)(\log_{3x} y) = \log_x x^3$, then y equals

(a) 125 (b) 25 (c) 5/3 (d) 243

47. The expression $\frac{(1+i)^n}{(1-i)^{n-2}}$ equals

(a) $-i^{n+1}$ (b) i^{n+1} (c) $-2i^{n+1}$ (d) 1

48. Let $z = x + iy$, where x and y are real. The points (x, y) in the X-Y plane for which $\frac{z+i}{z-i}$ purely imaginary lie on

(a) a straight line (b) an ellipse

(c) a hyperbola (d) a circle

49. If p, q are odd integers, then the roots of the equation $2px^2 + (2p+q)x + q = 0$ are

(a) rational (b) irrational

(c) non-real (d) equal

50. Out of 7 consonants and 4 vowels, words are formed each having 3 consonants and 2 vowels. The number of such words that can be formed is

(a) 210 (b) 25200 (c) 2520 (d) 302400

CATEGORY-II (Q. 51 to Q. 65)

Only one answer is correct. Correct answer will fetch full marks 2. Incorrect answer or any combination of more than one answer will fetch –½ marks. No answer will fetch 0 marks.

51. Let $A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$ Then for positive integer n, A^n is

(a) $\begin{pmatrix} 1 & n & n^2 \\ 0 & n^2 & n \\ 0 & 0 & n \end{pmatrix}$ (b) $\begin{pmatrix} 1 & n & n\left(\frac{n+1}{2}\right) \\ 0 & 1 & n \\ 0 & 0 & 1 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & n^2 & n \\ 0 & n & n^2 \\ 0 & 0 & n^2 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & n & 2n-1 \\ 0 & \frac{n+1}{2} & n^2 \\ 0 & 0 & \frac{n+1}{2} \end{pmatrix}$

52. Let a, b, c be such that $b(a+c) \neq 0$.

If $\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^n c \end{vmatrix} = 0$

then the value of n is

(a) any integer (b) zero

(c) any even integer (d) any odd integer

53. On set $A = \{1, 2, 3\}$, relations R and S are given by

$R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)\}$

$S = \{(1, 1), (2, 2), (3, 3), (1, 3), (3, 1)\}$. Then

(a) $R \cup S$ is an equivalence relation

(b) $R \cup S$ is reflexive and transitive but not symmetric

(c) $R \cup S$ is reflexive and symmetric but not transitive

(d) $R \cup S$ is symmetric and transitive but not reflexive

54. If one of the diameters of the curve $x^2 + y^2 - 4x - 6y + 9 = 0$ is a chord of a circle with centre (1, 1), the radius of the circle is

(a) 3 (b) 2 (c) $\sqrt{2}$ (d) 1

55. Let $A(-1, 0)$ and $B(2, 0)$ be two points. A point M moves in the plane in such a way that $\angle MBA = 2\angle MAB$. Then the point M moves along

(a) a straight line (b) a parabola

(c) an ellipse (d) a hyperbola

56. If $f(x) = \int_{-1}^{x} |t|\,dt$, then for any $x \geq 0$, $f(x)$ is equal to

(a) $\frac{1}{2}(1-x^2)$ (b) $1-x^2$

(c) $\frac{1}{2}(1+x^2)$ (d) $1+x^2$

57. Let for all $x > 0$, $f(x) = \lim_{n\to\infty} n\left(x^{\frac{1}{n}} - 1\right)$, then

(a) $f(x) + f\left(\frac{1}{x}\right) = 1$ (b) $f(xy) = f(x) + f(y)$

(c) $f(xy) = xf(y) + yf(x)$ (d) $f(xy) = xf(x) + yf(x)$

58. Let $I = \int_0^{100\pi} \sqrt{(1-\cos 2x)}\, dx$, then

(a) $I = 0$ (b) $I = 200\sqrt{2}$

(c) $I = \pi\sqrt{2}$ (d) $I = 100$

59. The area of the figure bounded by the parabolas $x = -2y^2$ and $x = 1 - 3y^2$ is

(a) $\frac{4}{3}$ square units (b) $\frac{2}{3}$ square units

(c) $\frac{3}{7}$ square units (d) $\frac{6}{7}$ square units

60. Tangents are drawn to the ellipse $\frac{x^2}{9} + \frac{y^2}{5} = 1$ at the ends of both latusrectum. The area of the quadrilateral so formed is

(a) 27 sq. units (b) $\frac{13}{2}$ sq. units

(c) $\frac{15}{4}$ sq. units (d) 45 sq. units

61. The value of K in order that $f(x) = \sin x - \cos x - Kx + 5$ decreases for all positive real values of x is given by

(a) $K < 1$ (b) $K \geq 1$ (c) $K > \sqrt{2}$ (d) $K < \sqrt{2}$

62. For any vector $\vec{x}$, the value of $(\vec{x} \times \hat{i})^2 + (\vec{x} \times \hat{j})^2 + (\vec{x} \times \hat{k})^2$ is equal to

(a) $|\vec{x}|^2$ (b) $2|\vec{x}|^2$ (c) $3|\vec{x}|^2$ (d) $4|\vec{x}|^2$

63. If the sum of two unit vectors is a unit vector, then the magnitude of their difference is

(a) $\sqrt{2}$ units (b) 2 units

(c) $\sqrt{3}$ units (d) $\sqrt{5}$ units

64. Let α and β be the roots of $x^2 + x + 1 = 0$. If n be positive integer, then $\alpha^n + \beta^n$ is

(a) $2\cos\frac{2n\pi}{3}$ (b) $2\sin\frac{2n\pi}{3}$

(c) $2\cos\frac{n\pi}{3}$ (d) $2\sin\frac{n\pi}{3}$

65. For real x, the greatest value of $\frac{x^2+2x+4}{2x^2+4x+9}$ is

(a) 1 (b) -1 (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

CATEGORY-III (Q. 66 to Q. 75)

One or more answer(s) is (are) correct. Correct answer(s) will fetch full marks 2. Any combination containing one or more incorrect answer will fetch 0 marks. Also no answer will fetch 0 marks. If all correct answers are not marked and also no incorrect answer is marked then score = 2 × number of correct answers marked ÷ actual number of correct answers.

66. If $a, b \in \{1, 2, 3\}$ and the equation $ax^2 + bx + 1 = 0$ has real roots, then

(a) $a > b$

(b) $a \leq b$

(c) number of possible ordered pairs (a, b) is 3

(d) $a < b$

67. If the tangent to $y^2 = 4ax$ at the point $(at^2, 2at)$ where $|t| > 1$ is a normal to $x^2 - y^2 = a^2$ at the point $(a\sec\theta, a\tan\theta)$, then

(a) $t = -\text{cosec}\,\theta$ (b) $t = -\sec\theta$

(c) $t = 2\tan\theta$ (d) $t = 2\cot\theta$

68. The focus of the conic $x^2 - 6x + 4y + 1 = 0$ is

(a) (2, 3) (b) (3, 2) (c) (3, 1) (d) (1, 4)

69. Let $f : R \to R$ be twice continuously. Let $f(0) = f(1) = f'(0) = 0$. Then

(a) $f''(x) \neq 0$ for all x

(b) $f''(c) \neq 0$ for some $c \in R$

(c) $f''(x) \neq 0$ if $x \neq 0$

(d) $f'(x) > 0$ for all x

70. If $f(x) = x^n$, n being a non-negative integer, then the values of n for which $f'(\alpha + \beta) = f'(\alpha) + f'(\beta)$ for all $\alpha, \beta > 0$ is

(a) 1 (b) 2 (c) 0 (d) 5

71. Let f be a non-constant continuous function for all $x \geq 0$. Let f satisfy the relation $f(x)\, f(a - x) = 1$ for some $a \in R^+$. Then $I = \int_0^a \frac{dx}{1+f(x)}$ is equal to

(a) a (b) $\frac{a}{4}$ (c) $\frac{a}{2}$ (d) $f(a)$

72. If the line $ax + by + c = 0$, $ab \neq 0$, is a tangent to the curve $xy = 1 - 2x$, then

(a) $a > 0, b < 0$ (b) $a > 0, b > 0$

(c) $a < 0, b > 0$ (d) $a < 0, b < 0$

73. Two particles move in the same straight line starting at the same moment from the same point in the same direction. The first moves with constant velocity u and the second starts from rest with constant acceleration f. Then

(a) they will be at the greatest distance at the end of time $\frac{u}{2f}$ from the start

(b) they will be at the greatest distance at the end of time $\frac{u}{f}$ from the start

(c) their greatest distance is $\frac{u^2}{2f}$

(d) their greatest distance is $\frac{u^2}{f}$

74. The complex number z satisfying the equation $|z-i| = |z+1| = 1$ is

(a) 0 (b) $1+i$ (c) $-1+i$ (d) $1-i$

75. On R, the set of real numbers, a relation ρ is defined as '$a\rho b$ if and only if $1 + ab > 0$. Then

(a) ρ is an equivalence relation

(b) ρ is reflexive and transitive but not symmetric

(c) ρ is reflexive and symmetric but not transitive

(d) ρ is only symmetric

SOLUTIONS

1. (b) : 1st digit must be other than 0 (zero) *i.e.* 9 ways.

$\therefore$ Other 4 digits can be filled in ${}^9P_4$ ways

$\therefore$ Total no. of numbers having 5 (distinct) digits $= 9 \times {}^9P_4$

2. (b) : $\because$ $(p+1)(p+2)(p+3).....(p+q)$ is a product of q consecutive positive integers

$\therefore$ It must be always divisible by $q!$

3. (b) : $(1+x+x^2)^9 = a_0 + a_1x + a_2x^2 + ... + a_{18}x^{18}$

Putting $x = 1$ and -1, we get

$3^9 = a_0 + a_1 + a_2 + ... + a_{18}$ ...(i)

$1 = a_0 - a_1 + a_2 - ..... + a_{18}$ ...(ii)

Adding (i) & (ii), we get

$$\frac{3^9+1}{2} = a_0 + a_2 + a_4 + ... + a_{18}$$

$\Rightarrow$ $a_0 + a_2 + a_4 + ... + a_{18} = 9842$, which is even but not divisible by 3 or 9.

4. (d) : $\because$ $\begin{vmatrix} 8 & -3 & -5 \\ 5 & -8 & 3 \\ 3 & 5 & -8 \end{vmatrix} = \begin{vmatrix} 0 & -3 & -5 \\ 0 & -8 & 3 \\ 0 & 5 & -8 \end{vmatrix}$

$[C_1 \to C_1 + C_2 + C_3]$

$= 0$

$\therefore$ Given system of equation has infinitely many non-zero solutions.

5. (b) : $\because$ Every orthogonal matrix is non-singular but every non-singular matrix may or may not be orthogonal.

$\therefore$ Q is proper subset of P.

6. (b) : $AB = \begin{pmatrix} x+2 & 3x \\ 3 & x+2 \end{pmatrix}\begin{pmatrix} x & 0 \\ 5 & x+2 \end{pmatrix}$

$= \begin{pmatrix} x^2+17x & 3x^2+6x \\ 8x+10 & x^2+4x+4 \end{pmatrix}$

$\det(AB) = x(x+2)\begin{vmatrix} x+17 & 3 \\ 8x+10 & x+2 \end{vmatrix}$

$= x(x+2)(x^2-5x+4) = x(x+2)(x-1)(x-4)$

$\therefore$ $\det(AB) = 0 \Rightarrow x = 1, 4, 0, -2$

7. (a) : $|A| = \begin{vmatrix} 1 & \cos\theta & 0 \\ -\cos\theta & 1 & \cos\theta \\ -1 & -\cos\theta & 1 \end{vmatrix} = (1+\cos^2\theta) - \cos\theta(0) + 0$

$= 1 + \cos^2\theta \in [1, 2]$.

8. (c) : $\because$ $f(x)\cdot f(y) = f(x+y)$ ...(i)

$\therefore$ $f(y)f(y) = f(y+y) \Rightarrow \{f(y)\}^2 = f(2y)$

$\Rightarrow$ $f(x)f(z) = f(2y)$ [$\because$ $f(x), f(y), f(z)$ are in G.P.]

$\Rightarrow$ $f(x+z) = f(2y)$ [from (i)]

$\Rightarrow$ x, y, z are in A.P. but will depend on the value of x, y, z.

9. (d) : $\because$ $x^2 \ge 0$ $\therefore$ $x\cdot x \ge 0 \Rightarrow xPx \Rightarrow$ Reflexive

$\because$ $xy \ge 0 \Rightarrow yx \ge 0 \Rightarrow$ Symmetric

$\because$ $(-5)(0) \ge 0$ & $(0)(7) \ge 0$

i.e., $(-5, 0) \in P$ & $(0, 7) \in P$

But, $(-5)(7) < 0 \Rightarrow (-5, 7) \notin P$

$\therefore$ P is not transitive.

10. (b) : Here $(x, y) \in \rho$ if $x - y$ is zero or irrational.

$\because$ $x - x = 0$ for all $x \in R$

$\Rightarrow$ ρ is reflexive

If $x - y$ is zero or irrational then $y - x$ is also zero or irrational.

$\Rightarrow$ ρ is symmetric.

Let $(x, y) \in \rho$ & $(y, z) \in \rho$

$\therefore$ $x - y = 0$ or irrational & $y - z = 0$ or irrational

But, their sum $x - z$ may or may not be 0 or irrational

e.g., $2 - \sqrt{3}$ is irrational & $\sqrt{3} - 5$ both are irrational but their sum $2 - 5 = -3$ is neither zero nor irrational

$\Rightarrow$ ρ is not transitive.

11. (d) : $\because$ Mean of n observations $x_1, x_2, ..., x_n$ is $\bar{x}$.

$\therefore$ Sum of n observations $= n\bar{x}$

If x_q is replaced by x_q' then sum $= n\bar{x} - x_q + x_q'$

$\therefore$ New mean $= \frac{n\bar{x} - x_q + x_q'}{n}$

12. (b) : A non-leap year has 52 weeks & 1 extra day

$\therefore$ Prob. of 53 sundays $= \frac{1}{7}$

13. (d) : We have, $k = \sin^2 x + \sin x \cos x = \frac{1}{2}[(1 - \cos 2x) + \sin 2x]$

$$= \frac{1}{2} + \frac{1}{\sqrt{2}}\left(\sin 2x \cdot \frac{1}{\sqrt{2}} - \cos 2x \cdot \frac{1}{\sqrt{2}}\right)$$

$$= \frac{1}{2} + \frac{1}{\sqrt{2}} \sin(2x - 45°)$$

$\because$ $-1 \le \sin(2x - 45°) \le 1$

$\therefore$ $\frac{1}{2} - \frac{1}{\sqrt{2}} \le k \le \frac{1}{2} + \frac{1}{\sqrt{2}}$

$\Rightarrow$ $\frac{1-\sqrt{2}}{2} \le k \le \frac{1+\sqrt{2}}{2}$

14. (a) : $\tan^{-1}\left(\frac{x+1}{x+2}\right) = \tan^{-1} 1 - \tan^{-1}\left(\frac{x-1}{x-2}\right)$

$\Rightarrow$ $\frac{x+1}{x+2} = \frac{1 - \frac{x-1}{x-2}}{1 + \frac{x-1}{x-2}}$ $\Rightarrow$ $\frac{x+1}{x+2} = \frac{-1}{2x-3}$

$\Rightarrow$ $2x^2 = 1$ $\Rightarrow$ $x = \pm\frac{1}{\sqrt{2}}$

15. (b) : Putting $x = x' + p$ and $y = y' + q$ in given equation, it becomes

$2(x' + p)^2 + 3(x' + p)(y' + q) + 4(y' + q)^2 + (x' + p) + 18(y' + q) + 25 = 0$

On comparing with $2x^2 + 3xy + 4y^2 = 1$, we get

$4p + 3q + 1 = 0$ ...(i)

$3p + 8q + 18 = 0$ ...(ii)

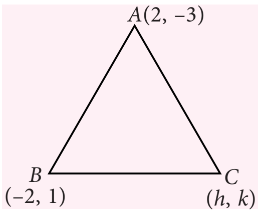
and $2p^2 + 3pq + 4q^2 + p + 18q + 25 = -1$ ...(iii)

On comparing (i) & (ii), $p = 2$, $q = -3$ by which (iii) is also satisfied.

16. (a) : Centroid $\equiv \left(\frac{2-2+h}{3}, \frac{-3+1+k}{3}\right)$

i.e., $\left(\frac{h}{3}, \frac{k-2}{3}\right)$

lies on $2x + 3y = 1$



$\therefore$ $2\left(\frac{h}{3}\right) + 3\left(\frac{k-2}{3}\right) = 1$

$\Rightarrow$ $2h + 3k - 6 = 3$

$\Rightarrow$ $2x + 3y = 9$ is the reqd. locus.

17. (d) : $P(3, 6)$ has reflection on $y = x$ as $Q(6, 3)$

Again reflection of $Q(6, 3)$ on $y = -x$ will be $Q'(-3, -6)$

$\because$ Slope of PQ × slope of $QQ' = (-1)(1) = -1$

$\therefore$ PQQ' is a right angled Δ with $\angle PQQ' = 90°$

$\therefore$ Circumcentre will be mid pt. of hypotenuse PQ' *i.e.* $(0, 0)$

18. (b) : $\because$ Bisectors of $3x + 4y = 5$ & $12x + 5y = 7$ are $\frac{3x+4y-5}{\sqrt{3^2+4^2}} = \pm\frac{12x+5y-7}{\sqrt{12^2+5^2}}$

$\Rightarrow$ $13(3x + 4y - 5) = \pm 5(12x + 5y - 7)$

$\Rightarrow$ $21x - 27y + 30 = 0$ & $99x + 77y = 100$

$\Rightarrow$ $7x - 9y + 10 = 0$ & $99x + 77y = 100$

$\because$ $7x - 9y + 10 = 0$ is one of the bisector.

$\therefore$ Perp. from any pt. on it will be equal to both given lines $\therefore$ $d_1 = d_2$.

19. (d) : Equation of common chord *i.e.*, $S_1 - S_2 = 0$ will be

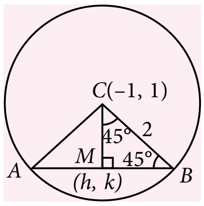
$\Rightarrow$ $(x^2 + y^2 - 16) - (x^2 + y^2 - 4x - 4y) = 0$

$\Rightarrow$ $x + y = 4$, which subtends 90° at (0, 0)

20. (c) : Let $M(h, k)$ be the mid point of chord of

$(x + 1)^2 + (y - 1)^2 = (2)^2$ ...(i)

subtending 90° at centre $C(-1, 1)$



$\therefore$ $2CM^2 = CB^2$

$\Rightarrow$ $(h + 1)^2 + (k - 1)^2 = (2)$

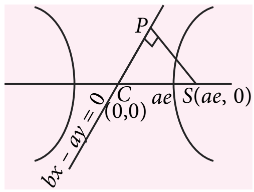
$\Rightarrow$ $x^2 + y^2 + 2x - 2y = 0$ is the reqd. locus.

21. (b) : $CS = ae$, $SP = \frac{b \cdot ae - 0}{\sqrt{b^2 + a^2}} = \frac{abe}{\sqrt{b^2 + a^2}}$

$\therefore$ $CP = \sqrt{CS^2 - SP^2}$

$$= \sqrt{a^2e^2 - \frac{b^2a^2e^2}{a^2 + b^2}}$$

$$= \frac{a^2 e}{\sqrt{a^2 + b^2}}$$



$\therefore$ Area of rectangle with sides SP and $CP = SP \cdot CP$

$$= \frac{ab \cdot a^2e^2}{a^2 + b^2} = ab \qquad [\because\ b^2 = a^2(e^2 - 1)]$$

22. (b) : $CB = b$, $CS = ae$

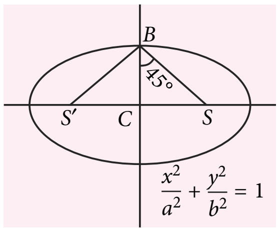
$\because$ $\angle SBS' = 90°$

$\therefore$ $CB = CS$

$\Rightarrow$ $b = ae$ $\Rightarrow$ $b^2 = a^2e^2$

$\Rightarrow$ $a^2(1 - e^2) = a^2e^2$

$\Rightarrow$ $2e^2 = 1$ $\therefore$ $e = \frac{1}{\sqrt{2}}$



23. (a) : Parabola is $(x + y)^2 = 5(x - y + 1)$ whose axis is $x + y = 0$.

24. (c) : Foci of hyperbola $y^2 - x^2 = 1$ are

$(0, \pm be)$ *i.e.*, $(0, \pm\sqrt{2})$

$\therefore$ End points of a diameter of reqd. circle are $(0, \sqrt{2})$ and $(0, -\sqrt{2})$.

$\therefore$ Eqn. of reqd. circle is

$(x - 0)(x - 0) + (y - \sqrt{2})(y + \sqrt{2}) = 0$

or $x^2 + y^2 = 2$

25. (d) : Any plane through $(1, 2, -3)$ is given by

$a(x - 1) + b(y - 2) + c(z + 3) = 0$ ...(i)

If (i) is || to x-axis then $a = 0$

If (i) passes through $(2, -2, 1)$ then

$b(-2 - 2) + c(1 + 3) = 0 \Rightarrow b = c$

$\therefore$ (i) becomes $b(y - 2) + b(z + 3) = 0$

$\Rightarrow y + z + 1 = 0$

26. (b) : Here, $\begin{vmatrix} 1 & -1 & 1 \\ 2 & -3 & 0 \\ 1 & 0 & 3 \end{vmatrix} = 1(-9-0)+1(6-0)+1(0+3) = -9+6+3=0$

$\therefore$ 3 lines with d.r.s $(1, -1, 1)$, $(2, -3, 0)$ & $(1, 0, 3)$ are coplanar.

27. (a) : Given, $f(x)$ is non-constant & differentiable s.t. $\dfrac{f(x)}{f(y)} = f(x - y)$ ...(i)

Let $f(x) = a^{mx}$ satisfying (i). Now, $f'(x) = ma^{mx}$

Given $f'(0) = p \Rightarrow ma^0 = p \Rightarrow m = p$ ...(ii)

Also, $f'(5) = q \Rightarrow ma^{5m} = q$

$\Rightarrow a^{5m} = \dfrac{q}{m} = \dfrac{q}{p}$ ...(iii)

$\therefore f'(-5) = ma^{m(-5)} = p \cdot a^{-5m} = p \cdot \dfrac{p}{q}$ [using (iii)]

$= \dfrac{p^2}{q}$

28. (c) : $f(x) = \log_5 \log_3 x = \log_e(\log_3 x)\log_5 e$

$= \log_5 e \log_e(\log_3 x)$

$$f'(x) = \log_5 e \frac{1}{\log_3 x} \cdot \frac{d}{dx}(\log_3 x)$$

$$= \log_5 e \cdot \frac{1}{\log_3 x} \frac{d}{dx}(\log_e x \cdot \log_3 e)$$

$$= \log_5 e \cdot \frac{1}{\log_3 x} \cdot \log_3 e \cdot \frac{1}{x} = \log_5 e \log_x e \cdot \frac{1}{x}$$

$$\therefore f'(e) = \log_5 e \log_e e \cdot \frac{1}{e} = \frac{1}{e \log_e 5}$$

29. (c) : $H(x) = G(F(x)) = G(e^x) = e^{-e^x}$

$$\therefore \frac{dH}{dx} = e^{-e^x} \cdot \frac{d}{dx}(-e^x) = e^{-e^x} \cdot (-e^x)$$

$$\therefore \left[\frac{dH}{dx}\right]_{x=0} = e^{-e^0}(-e^0) = -e^{-1} = -\frac{1}{e}$$

30. (c) : $\displaystyle \lim_{x\to 0} \frac{2f(x) - 3f(2x) + f(4x)}{x^2}$ $\left(\frac{0}{0}\text{ form}\right)$

$\displaystyle = \lim_{x\to 0} \frac{2f'(x) - 6f'(2x) + 4f'(4x)}{2x}$ $\left(\frac{0}{0}\text{ form}\right)$

$\displaystyle = \lim_{x\to 0} \frac{2f''(x) - 12f''(2x) + 16f''(4x)}{2}$

$= f''(0) - 6f''(0) + 8f''(0)$

$= 3f''(0) = 3k$

31. (a) : We have, $y = e^{m\sin^{-1}x}$

$$\Rightarrow \frac{dy}{dx} = e^{m\sin^{-1}x} \cdot m \cdot \frac{1}{\sqrt{1-x^2}} = \frac{my}{\sqrt{1-x^2}}$$

$$\Rightarrow (1-x^2)\left(\frac{dy}{dx}\right)^2 = m^2y^2$$

$$\Rightarrow (1-x^2) \cdot 2\frac{dy}{dx} \cdot \frac{d^2y}{dx^2} - 2x \cdot \left(\frac{dy}{dx}\right)^2 = m^2 \cdot 2y \cdot \frac{dy}{dx}$$

$$\Rightarrow (1-x^2)\left(\frac{d^2y}{dx^2}\right) - x \cdot \frac{dy}{dx} = m^2 y \Rightarrow k = m^2$$

32. (d) : Using mean value theorem, $f'(c) = \dfrac{f(b) - f(a)}{b - a}$

$$\Rightarrow 2c + 2a = \frac{f(\beta) - f(\alpha)}{\beta - \alpha}$$

$$= \frac{(\beta^2 + 2a\beta + b) - (\alpha^2 + 2a\alpha + b)}{\beta - \alpha} = \beta + \alpha + 2a$$

$$\therefore c = \frac{\alpha + \beta}{2}$$

33. (c) : $f(x) = x^{13} + x^{11} + x^9 + x^7 + x^5 + x^3 + x + 19$

$f'(x) = 13x^{12} + 11x^{10} + 9x^8 + 7x^6 + 5x^4$
$+ 3x^2 + 1 > 0 \ \forall\ x \in R$

$\therefore$ $f(x)$ is a strictly increasing function.

$f(-\infty) = -\infty$, $f(\infty) = \infty$, $f(0) = 19$

$\therefore$ $f(x) = 0$, will have only one real root

34. (b) : $\because$ LMVT is applicable to $f(x)$ in $[0, x]$

$\therefore$ $f(x)$ must be continuous in $[0, x]$

$$\therefore \lim_{x\to 0^+} f(x) = f(0) \Rightarrow \lim_{x\to 0^+} \frac{x^p}{(\sin x)^q} = 0$$

$$\Rightarrow \lim_{h\to 0} \frac{(0+h)^p}{\{\sin(0+h)\}^q} = 0 \Rightarrow \lim_{h\to 0} \frac{h^{p-q}}{\left(\frac{\sin h}{h}\right)^q} = 0$$

$$\Rightarrow \lim_{h\to 0} \frac{h^{p-q}}{1} = 0$$ $\therefore$ L.H.L exists only if $p > q$.

35. (b) : Let $A = \displaystyle\lim_{x\to 0} (\sin x)^{2\tan x}$

$$\log A = \lim_{x\to 0} 2\tan x \log(\sin x)$$

$$= 2\lim_{x\to 0} \frac{\log(\sin x)}{\cot x} \quad \left(\frac{\infty}{\infty}\text{ form}\right)$$

$$= 2\lim_{x\to 0} \frac{\frac{1}{\sin x} \cdot \cos x}{-\operatorname{cosec}^2 x} = -\lim_{x\to 0} \sin 2x = 0 \Rightarrow A = 1$$

36. (c) : Let $I = \displaystyle\int \cos(\log x)dx$

$$= \cos(\log x) \cdot (x) - \int -\sin(\log x) \cdot \frac{1}{x} \cdot (x)dx$$

$$= x\cos(\log x)+\int \sin(\log x)dx$$

$$= x\cos(\log x) + \sin(\log x)(x) - \int \cos(\log x)\cdot\frac{1}{x}(x)dx$$

$$\Rightarrow \quad 2I = x[\cos(\log x) + \sin(\log x)]$$

$$\Rightarrow \quad I = \frac{x}{2}[\cos(\log x) + \sin(\log x)] + c$$

37. (a) : $\displaystyle\int \frac{x^2\left(1-\frac{1}{x^2}\right)dx}{x^2\left(x^2+\frac{1}{x^2}+3\right)} = \int \frac{\left(1-\frac{1}{x^2}\right)dx}{\left(x+\frac{1}{x}\right)^2+1}$

$$= \int \frac{dz}{z^2+1}, \text{ where } z = x+\frac{1}{x}$$

$$= \tan^{-1}z + c = \tan^{-1}\left(x+\frac{1}{x}\right)+c$$

38. (b) : $\because \quad I = \displaystyle\int_{10}^{19}\frac{\sin x}{1+x^8}dx$

$$\therefore \quad |I| = \left|\int_{10}^{19}\frac{\sin x}{1+x^8}dx\right| \le \int_{10}^{19}\left|\frac{\sin x}{1+x^8}\right|dx$$

$$\Rightarrow \quad |I| \le \int_{10}^{19}\frac{|\sin x|}{1+x^8}dx \le \int_{10}^{19}\frac{dx}{1+x^8} \qquad [\because |\sin x|\le 1]$$

$$\Rightarrow \quad |I| \le \int_{10}^{19}\frac{dx}{x^8} = \left[\frac{x^{-7}}{-7}\right]_{10}^{19}$$

$$= \frac{1}{7}(10^{-7}-19^{-7}) < \frac{10^{-7}}{7} < 10^{-7}$$

39. (d) : $I_2 = \displaystyle\int_0^n \{x\}dx = \int_0^{n\cdot 1}\{x\}dx = n\int_0^1\{x\}dx$

$[\because \quad \{x\}$ is periodic with period $= 1]$

$$= n\int_0^1 x\,dx = \frac{n}{2}$$

$$I_1 + I_2 = \int_0^n([x]+\{x\})dx = \int_0^n x\,dx = \left[\frac{x^2}{2}\right]_0^n = \frac{n^2}{2}$$

$$\therefore \quad I_1 = \frac{n^2}{2}-\frac{n}{2} \quad \therefore \quad \frac{I_1}{I_2} = \frac{n^2-n}{n} = n-1$$

40. (b) : $\displaystyle\lim_{n\to\infty}\left[\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+.....+\frac{n}{n^2+n^2}\right]$

$$= \lim_{n\to\infty}\frac{1}{n}\left[\frac{1}{1+\left(\frac{1}{n}\right)^2}+\frac{1}{1+\left(\frac{2}{n}\right)^2}+.....+\frac{1}{1+\left(\frac{n}{n}\right)^2}\right]$$

$$= \lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\frac{1}{1+\left(\frac{r}{n}\right)^2} = \int_0^1\frac{dx}{1+x^2} = [\tan^{-1}x]_0^1 = \frac{\pi}{4}$$

41. (d) : $\because \quad 0 \le x \le 1 \quad \therefore \quad 1 \le e^{x^2} \le e$

$$\Rightarrow \quad \int_0^1 dx \le \int_0^1 e^{x^2}dx \le \int_0^1 e\cdot dx$$

$$\Rightarrow \quad 1 \le \int_0^1 e^{x^2}dx \le e$$

42. (c) : $\displaystyle\int_0^{100(1)} e^{x-[x]}dx = \int_0^{100(1)} e^{\{x\}}dx = 100\int_0^1 e^{\{x\}}dx$

$[\because \quad \{x\}$ is periodic with period $1]$

$$= 100\int_0^1 e^x dx = 100(e-1)$$

43. (a) : Putting $x + y = z \Rightarrow 1+\dfrac{dy}{dx} = \dfrac{dz}{dx}$

Given eq. becomes $z^2\cdot\left(\dfrac{dz}{dx}-1\right) = a^2$

$$\Rightarrow \quad \frac{dz}{dx} = 1+\frac{a^2}{z^2}$$

$$\Rightarrow \quad \int\frac{z^2}{z^2+a^2}dz = \int dx \quad \Rightarrow \quad \int\frac{(z^2+a^2)-a^2}{z^2+a^2}dz = x$$

$$\Rightarrow \quad z - a^2\cdot\frac{1}{a}\tan^{-1}\frac{z}{a} = x - c$$

$$\Rightarrow \quad x + y - a\tan^{-1}\left(\frac{x+y}{a}\right) = x - c$$

$$\Rightarrow \quad \frac{y+c}{a} = \tan^{-1}\left(\frac{x+y}{a}\right) \quad \Rightarrow \quad \frac{x+y}{a} = \tan\left(\frac{y+c}{a}\right),$$

c is an arbitrary constant.

44. (b) : We have, $\dfrac{dy}{dx}+\dfrac{x(x^2+1)}{x^2(x^2-1)}\cdot y = \dfrac{x^2-1}{x^2(x^2-1)}$

$$\Rightarrow \quad \frac{dy}{dx}+\frac{x^2+1}{x(x^2-1)}\cdot y = \frac{1}{x^2}$$

$$\therefore \quad \text{I.F.} = e^{\int\frac{x^2+1}{x(x^2-1)}dx} = e^{\int\frac{\left(1+\frac{1}{x^2}\right)}{x-\frac{1}{x}}dx} = e^{\log_e\left(x-\frac{1}{x}\right)} = x-\frac{1}{x}$$

45. (b) : Let G.P. be $a, ar, ar^2, ...$

A.T.Q., $a = ar + ar^2 \quad \Rightarrow \quad r^2 + r - 1 = 0$

$$\Rightarrow \quad r = \frac{-1\pm\sqrt{1+4}}{2} = \frac{-1\pm\sqrt{5}}{2}$$

$\because$ Terms are positive $\quad\therefore\quad r \not< 0 \quad \therefore \quad r = \dfrac{-1+\sqrt{5}}{2}$

46. (a) : Using property of logarithm, $\log_5 y = 3\log_x x = 3$

$\therefore \quad y = 5^3 = 125$

47. (c) : $\left(\frac{1+i}{1-i}\right)^n (1-i)^2 = \left\{\frac{(1+i)^2}{1-i^2}\right\}^n (1+i^2-2i)$

$= \left\{\frac{1-1+2i}{2}\right\}^n (-2i) = -2i^{n+1}$

48. (d) : Let $\frac{z+i}{z-i} = ki$ $\quad (k \in R)$

By componendo & dividendo, we have $\frac{2z}{2i} = \frac{ki+1}{ki-1}$

$\Rightarrow \quad z = \frac{ki+1}{ki-1}.i \Rightarrow |z| = \frac{\sqrt{k^2+1^2}}{\sqrt{k^2+(-1)^2}}.|i| = 1$

$\Rightarrow \quad x^2 + y^2 = 1$ which represents a circle

49. (a) : $2px^2 + 2px + qx + q = 0$

$\Rightarrow \quad 2px(x + 1) + q(x + 1) = 0$

$\therefore \quad x = -1, \frac{-q}{2p}$, which are rational as p & q are odd integers.

50. (b) : 3 out of 7 consonants can be chosen in ${}^7C_3$ ways and 12 out of 4 vowels can be chosen in ${}^4C_2$ ways

$\therefore$ Total no. of words that can be formed $= {}^7C_3 \times {}^4C_2 \times \underline{|5}$
$= 25200$

51. (b) : $A^2 = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$

Only (b) is satisfied by putting $n = 2$

52. (d) : $D_1 = \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} = \begin{vmatrix} a & -b & c \\ a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \end{vmatrix}$

[Interchanging rows & columns]

$= (-1)^2 \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ a & -b & c \end{vmatrix} = \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ a & -b & c \end{vmatrix}$

$D_2 = \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^n c \end{vmatrix}$

$= (-1)^n \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ a & -b & c \end{vmatrix}$

$\therefore \quad D_1 + D_2 = 0$ is possible only when n is any odd integer.

53. (c): $R \cup S = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (1, 3), (3, 1)\}$

$A = \{1, 2, 3\}$

(i) $\because$ $(1, 1), (2, 2), (3, 3) \in R \cup S$ $\Rightarrow$ Reflexive

(ii) $(a, b) \in R \cup S$

$\Rightarrow$ $(b, a) \in R \cup S \ \forall \ a, b \ \{1, 2, 3\}$ $\Rightarrow$ Symmetric

(iii) $\because$ (2, 1) & (1, 3)$\in R \cup S$ but $(2, 3) \notin R \cup S$

$\Rightarrow$ Not transitive

54. (a) : Let radius of circle with centre (1, 1) be a

Its eqn. is $(x - 1)^2 + (y - 1)^2 = a^2$ ...(i)

Given circle is $x^2 + y^2 - 4x - 6y + 9 = 0$ ...(ii)

Eqn. of common chord $(S_1 - S_2 = 0)$ is

$2x + 4y = a^2 + 7$ ...(iii)

If (iii) be a diameter of (ii) then centre (2, 3) will lie on (iii)

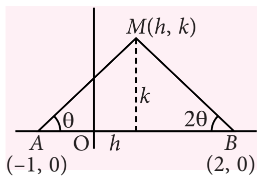
$\Rightarrow \quad 4 + 12 = a^2 + 7 \quad \Rightarrow \quad a^2 = 9$

$\Rightarrow$ Radius = 3 units

55. (d) : $\tan\theta = \frac{k}{h+1}$ & $\tan 2\theta = \frac{k}{2-h}$

$\Rightarrow \quad \frac{k}{2-h} = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\left(\frac{k}{h+1}\right)}{1-\left(\frac{k}{h+1}\right)^2}$

$= \frac{2k(h+1)}{(h+1)^2-k^2}$



$\Rightarrow \quad (h + 1)^2 - k^2$

$= 2(2 - h)(h + 1)$

$\Rightarrow \quad h^2 + 2h + 1 - k^2$

$= 4h + 4 - 2h^2 - 2h$

$\Rightarrow \quad 3h^2 - k^2 = 3 \quad \Rightarrow \quad \frac{h^2}{1} - \frac{k^2}{3} = 1$

$\therefore \quad M$ moves on a hyperbola

56. (c) : $f(x) = \int_{-1}^{x} |t|\,dt = \int_{-1}^{0} |t|\,dt + \int_{0}^{x} |t|\,dt$

$= \int_{-1}^{0} |x|\,dx + \int_{0}^{x} |x|\,dx = \int_{-1}^{0} (-x)\,dx + \int_{0}^{x} x\,dx$

$= \left[-\frac{x^2}{2}\right]_{-1}^{0} + \left[\frac{x^2}{2}\right]_{0}^{x} = -\frac{1}{2}(0-1) + \left(\frac{x^2}{2} - 0\right) = \frac{x^2+1}{2}$

57. (b) : $f(x) = \lim_{n\to\infty} n\left(x^{\frac{1}{n}} - 1\right) = \lim_{k\to 0} \frac{x^k-1}{k}$, where $n = \frac{1}{k}$

$= \log_e x$

Now, $f(x) + f\left(\frac{1}{x}\right) = \log_e x + \log_e 1/x = \log_e 1 = 0$

$\therefore \quad f(xy) = \log_e xy = \log_e x + \log_e y = f(x) + f(y)$

58. (b) : $I = \int_0^{100\pi} \sqrt{2\sin^2 x}\,dx$

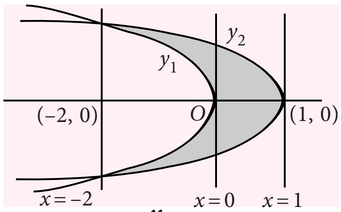
$= \sqrt{2}\int_0^{100\pi}|\sin x|\,dx = \sqrt{2}\cdot 100\int_0^{\pi}|\sin x|\,dx$

[$\because$ Period of $|\sin x|$ is π]

$= 100\sqrt{2}\int_0^{\pi}\sin x\,dx = 100\sqrt{2}[-\cos x]_0^{\pi}$

$= 100\sqrt{2}(1+1) = 200\sqrt{2}$

59. (a) :



Parabolas are $y^2 = -\frac{x}{2}$ ...(i) & $y^2 = -\frac{1}{3}(x-1)$ ...(ii)

On solving, $-\frac{x}{2} = -\frac{(x-1)}{3}$

$\Rightarrow$ $-2x + 2 = -3x$ $\Rightarrow$ $x = -2$

$\therefore$ Reqd. area $= 2\left[\int_{-2}^{1} y_2 dx - \int_{-2}^{0} y_1 dx\right]$

[y_1, y_2 are values of y from (i) & (ii) resp.]

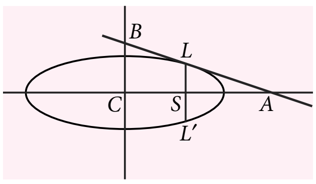
$= 2\left[\int_{-2}^{1}\sqrt{\frac{1-x}{3}}\,dx - \int_{-2}^{0}\sqrt{-\frac{x}{2}}\,dx\right]$

$= \frac{2}{\sqrt{3}}\left[\frac{(1-x)^{3/2}}{\left(-\frac{3}{2}\right)}\right]_{-2}^{1} - \frac{2}{\sqrt{2}}\left[\frac{(-x)^{3/2}}{\left(-\frac{3}{2}\right)}\right]_{-2}^{0}$

$= -\frac{4}{3\sqrt{3}}[0-3^{3/2}] + \frac{4}{3\sqrt{2}}[0-2^{3/2}]$

$= 4 - \frac{8}{3} = \frac{4}{3}$ square units

60. (a) : We have,



$\frac{x^2}{9} + \frac{y^2}{5} = 1$ ...(i)

$e = \sqrt{1-\frac{5}{9}} = \frac{2}{3}$

$\therefore$ $L\left(ae, \frac{b^2}{a}\right) \equiv \left(2, \frac{5}{3}\right)$

Eqn. of tangent to (i) at $L\left(2, \frac{5}{3}\right)$ is

$\frac{x}{9}\cdot 2 + \frac{y}{5}\cdot\frac{5}{3} = 1$ $\Rightarrow$ $\frac{x}{9/2} + \frac{y}{3} = 1$

$\therefore$ $CA = \frac{9}{2}, CB = 3$

$\therefore$ Reqd. area of quad. $= 4 \times$ Area of ΔCAB

$= 4\times\frac{1}{2}\cdot\frac{9}{2}\cdot 3 = 27$ sq. units.

61. (c) : $f(x) = \sin x - \cos x - Kx + 5$

$\Rightarrow$ $f'(x) = \cos x + \sin x - K = \sqrt{2}\sin\left(x+\frac{\pi}{4}\right) - K$

$\because$ $\sqrt{2}\sin\left(x+\frac{\pi}{4}\right) \le \sqrt{2}$

$\therefore$ $f(x)$ will be decreasing for all +ve real x if $f'(x) < 0$

$\sqrt{2}\sin\left(x+\frac{\pi}{4}\right) - K < 0$

$\Rightarrow$ $K > \sqrt{2}\sin\left(x+\frac{\pi}{4}\right)$ $\Rightarrow$ $K > \sqrt{2}$

62. (b) : Let $\vec{x} = a\hat{i} + b\hat{j} + c\hat{k}$

$\therefore$ $\vec{x}\times\hat{i} = -b\hat{k} + c\hat{j}$

$(\vec{x}\times\hat{i})^2 = |\vec{x}\times\hat{i}|^2 = b^2 + c^2$

Similarly, $(\vec{x}\times\hat{j})^2 = c^2 + a^2$ & $(\vec{x}\times\hat{k})^2 = a^2 + b^2$

$\therefore$ $(\vec{x}\times\hat{i})^2 + (\vec{x}\times\hat{j})^2 + (\vec{x}\times\hat{k})^2 = 2(a^2+b^2+c^2) = 2|\vec{x}|^2$

63. (c) : Let $\vec{a}, \vec{b}, \vec{c}$ be three unit vectors such that $\vec{c} = \vec{a} + \vec{b}$

$\therefore$ $\vec{c}^{\,2} = (\vec{a}+\vec{b})^2$ $\Rightarrow$ $1 = 1 + 1 + 2\vec{a}\cdot\vec{b}$

$\Rightarrow$ $2\,\vec{a}\cdot\vec{b} = -1$

Now, $|\vec{a}-\vec{b}|^2 = 1 + 1 - 2\vec{a}\cdot\vec{b} = 3 \Rightarrow |\vec{a}-\vec{b}| = \sqrt{3}$ units.

64. (a) : $\because$ Roots of $x^2 + x + 1 = 0$ are ω & ω^2

Let $\alpha = \omega = \frac{-1+\sqrt{3}i}{2} = \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$

and $\beta = \omega^2 = \frac{-1-\sqrt{3}i}{2} = \cos\left(-\frac{2\pi}{3}\right) + i\sin\left(-\frac{2\pi}{3}\right)$

$= \cos\frac{2\pi}{3} - i\sin\frac{2\pi}{3}$ $\therefore$ $\alpha^n + \beta^n = 2\cos\frac{2n\pi}{3}$

65. (c) : Let $y = \frac{x^2+2x+4}{2x^2+4x+9}$

$\Rightarrow$ $(2y-1)x^2 + 2(2y-1)x + (9y-4) = 0$

For real x, $D \ge 0$ $\Rightarrow$ $4(2y-1)^2 - 4(2y-1)(9y-4) \ge 0$

$\Rightarrow$ $4y^2 - 4y + 1 - 18y^2 + 17y - 4 \ge 0 \Rightarrow 14y^2 - 13y + 3 \le 0$

$\Rightarrow$ $(7y-3)(2y-1) \le 0$ $\Rightarrow$ $\frac{3}{7} \le y \le \frac{1}{2}$

$\Rightarrow$ Greatest value of y *i.e.* $\frac{x^2+2x+4}{2x^2+4x+9}$ is $\frac{1}{2}$

66. (c, d) : $\because$ $a, b \in \{1, 2, 3\}$ & $ax^2 + bx + 1 = 0$ has real roots

$\Rightarrow$ $D \ge 0$ *i.e.*, $b^2 \ge 4a$ ...(i)

$\therefore$ Possible ordered pair (a, b) are $(1, 2)$, $(1, 3)$ & $(2, 3)$ only

67. (a, c) : Equation of tangent to $y^2 = 4ax$ at $(at^2, 2at)$ is $y \cdot 2at = 2a\,(x + at^2)$

$\Rightarrow \quad ty = x + at^2$ ...(i)

For $x^2 - y^2 = a^2$, $\dfrac{dy}{dx} = \dfrac{x}{y}$

$\Rightarrow$ At $(a \sec\theta, a \tan\theta)$, $\dfrac{dy}{dx} = \dfrac{a\sec\theta}{a\tan\theta} = \dfrac{1}{\sin\theta}$

$\therefore$ Eqn. of normal to $x^2 - y^2 = a^2$ at $(a \sec\theta, a \tan\theta)$ is $y - a\tan\theta = -\sin\theta\,(x - a\sec\theta)$

$\Rightarrow \quad y = -x\sin\theta + 2a\tan\theta$ ...(ii)

$\because$ (i) & (ii) are identical

$\therefore \quad \dfrac{t}{1} = \dfrac{1}{-\sin\theta} = \dfrac{at^2}{2a\tan\theta} = \dfrac{t^2}{2\tan\theta}$

$\Rightarrow \quad t = -\text{cosec}\,\theta \quad \& \quad t = 2\tan\theta$

68. (c) : Conic may be written as $(x - 3)^2 = -4(y - 2)$

$X^2 = 4AY$ ...(i)

Here, $X = x - 3$, $Y = y - 2$ & $4A = -4$ ...(ii)

For focus, $X = 0$, $Y = A \quad \Rightarrow \quad x - 3 = 0$, $y - 2 = -1$

$\Rightarrow \quad x = 3, y = 1 \quad \therefore \quad$ Focus is $(3, 1)$

69. (b)

70. (b,c) : $f(x) = x^n$ ...(i) $f'(\alpha + \beta) = f'(\alpha) + f'(\beta)$ ...(ii)

When $n = 2$, $f(x) = x^2$, $f'(x) = 2x$

$\therefore \quad f'(\alpha + \beta) = 2(\alpha + \beta) = 2\alpha + 2\beta = f'(\alpha) + f'(\beta)$

$\Rightarrow$ (ii) is satisfied

When $n = 0$, $f(x) = 1 \Rightarrow f'(x) = 0 \Rightarrow$ (ii) is satisfied.

(ii) is not satisfied if $n = 1$ or 5

71. (c) : $I = \displaystyle\int_0^a \frac{dx}{1 + f(x)}$ ...(i)

$$= \int_0^a \frac{dx}{1 + f(a - x)} = \int_0^a \frac{f(x) f(a - x)}{f(x) f(a - x) + f(a - x)} dx$$

$$I = \int_0^a \frac{f(x)}{f(x) + 1} dx \qquad \text{...(ii)}$$

Adding (i) & (ii), $2I = \displaystyle\int_0^a dx = a \quad \Rightarrow \quad I = \frac{a}{2}$

72. (b,d) : $ax + by + c = 0$ ...(i)

Its slope $= \dfrac{-a}{b}$ $(ab \neq 0)$

Now, $xy = 1 - 2x \quad \Rightarrow \quad y + 2 = \dfrac{1}{x}$ ...(ii)

Differentiating (ii), we get, $\dfrac{dy}{dx} = -\dfrac{1}{x^2}$

$\therefore \quad \dfrac{dy}{dx} < 0$

If (i) be a tangent to (ii) then $-\dfrac{a}{b} = -\dfrac{1}{x^2} < 0$

$\Rightarrow \quad \dfrac{b}{a} = x^2 > 0$

$\therefore \quad a$ & b must be of same sign

$\therefore \quad a > 0, b > 0$ or $a < 0, b < 0$

73. (b,c) : $\vec{u}$ (at $\vec{0}$), $f = 0$ (at f)

Dist. between them at any instant is given by

$$x = u \cdot t - \left\{0 \cdot t + \frac{1}{2} f t^2\right\}$$

$$\frac{dx}{dt} = u - \frac{1}{2} \cdot f \cdot 2t = u - ft \Rightarrow \frac{d^2x}{dt^2} = -f < 0$$

For max. distance between them, $\dfrac{dx}{dt} = 0 \quad \Rightarrow \quad t = \dfrac{u}{f}$

Max. distance $= u \cdot \dfrac{u}{f} - \dfrac{1}{2} f \cdot \dfrac{u^2}{f^2} = \dfrac{u^2}{f} - \dfrac{u^2}{2f} = \dfrac{u^2}{2f}$

74. (a, c) : $|z - i| = |z + 1| = 1$

$\Rightarrow \quad x^2 + (y - 1)^2 = (x + 1)^2 + y^2 = 1$ ...(i)

$\Rightarrow \quad -2y + 1 = 2x + 1 \quad \Rightarrow \quad y = -x$ ...(ii)

When $y = -x$, we have $x^2 + 2x + 1 + x^2 = 1$

$\Rightarrow \quad 2x(x + 1) = 0 \quad \Rightarrow \quad x = 0, -1$

when $x = 0 \quad \Rightarrow \quad y = 0$

when $x = -1 \quad \Rightarrow \quad y = 1$

$\therefore \quad z = 0, -1 + i$

75. (c) : (i) $\because$ $1 + a \cdot a = 1 + a^2 > 0 \Rightarrow$ Reflexive

(ii) If $1 + ab > 0$ then $1 + ba > 0 \Rightarrow$ Symmetric

(iii) $\because 1 + 1\left(\dfrac{1}{2}\right) = \dfrac{3}{2} > 0 \Rightarrow \left(1, \dfrac{1}{2}\right) \in \rho$

$1 + \dfrac{1}{2}(-1) = \dfrac{1}{2} > 0 \Rightarrow \left(\dfrac{1}{2}, -1\right) \in \rho$

But, $1 + (1)(-1) = 0 \not> 0$

$\Rightarrow \quad (1, -1) \notin \rho$

$\therefore \quad \rho$ is not transitive.

SOLVED PAPER **2017**

Kerala PET

1. $\begin{vmatrix} 1 & 1 & 1 \\ p & q & r \\ p & q & r+1 \end{vmatrix}$ is equal to

(a) $q - p$ (b) $q + p$ (c) q (d) p
(e) 0

2. Let $A = \begin{bmatrix} 5 & 0 \\ 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$.

If $4A + 5B - C = O$, then C is

(a) $\begin{bmatrix} 5 & 25 \\ -1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 20 & 5 \\ -1 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 5 & -1 \\ 0 & 25 \end{bmatrix}$ (d) $\begin{bmatrix} 5 & 25 \\ -1 & 5 \end{bmatrix}$

(e) $\begin{bmatrix} 0 & 5 \\ 5 & 25 \end{bmatrix}$

3. If $U = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$, then U^{-1} is

(a) U^T (b) U (c) I (d) 0
(e) U^2

4. If $A = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$, then A^{-1} is

(a) A^T (b) A^2 (c) A (d) I
(e) O

5. If $\begin{pmatrix} x+y & x-y \\ 2x+z & x+z \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}$, then the values of x, y and z are respectively

(a) 0, 0, 1 (b) 1, 1, 0 (c) −1, 0, 0 (d) 0, 0, 0
(e) 1, 1, 1

6. $\begin{pmatrix} 7 & 1 & 5 \\ 8 & 0 & 0 \end{pmatrix}\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} + 5\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ is equal to

(a) $\begin{pmatrix} 16 \\ 27 \end{pmatrix}$ (b) $\begin{pmatrix} 27 \\ 16 \end{pmatrix}$ (c) $\begin{pmatrix} 15 \\ 16 \end{pmatrix}$ (d) $\begin{pmatrix} 16 \\ 15 \end{pmatrix}$

(e) $\begin{pmatrix} 16 \\ 16 \end{pmatrix}$

7. If $\begin{pmatrix} 1 & 2 & 4 \\ 1 & 3 & 5 \\ 1 & 4 & a \end{pmatrix}$ is singular, then the value of a is

(a) $a = -6$ (b) $a = 5$
(c) $a = -5$ (d) $a = 6$
(e) $a = 0$

8. If $\begin{pmatrix} 1 & 2 & -3 \\ 0 & 4 & 5 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, then (x, y, z) is equal to

(a) (1, 6, 6) (b) (1, −6, 1)
(c) (1, 1, 6) (d) (6, −1, 1)
(e) (−1, 6, 1)

9. If $A = \begin{pmatrix} 1 & 5 \\ 0 & 2 \end{pmatrix}$, then

(a) $A^2 - 2A + 2I = O$ (b) $A^2 - 3A + 2I = O$
(c) $A^2 - 5A + 2I = O$ (d) $2A^2 - A + I = O$
(e) $A^2 + 3A + 2I = O$

10. If $\begin{pmatrix} 2x+y & x+y \\ p-q & p+q \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$, then (x, y, p, q) equals

(a) 0, 1, 0, 0 (b) 0, −1, 0, 0
(c) 1, 0, 0, 0 (d) 0, 1, 0, 1
(e) 1, 0, 1, 0

11. The value of $\left|\sqrt{4+2\sqrt{3}}\right|-\left|\sqrt{4-2\sqrt{3}}\right|$ is

(a) 1 (b) 2 (c) 4 (d) 3
(e) 5

12. The value of $8^{2/3}-16^{1/4}-9^{1/2}$ is

(a) −1 (b) −2 (c) −3 (d) −4
(e) −5

13. Let $x = 2$ be a root of $y = 4x^2 - 14x + q = 0$. Then y is equal to

(a) $(x - 2)(4x - 6)$ (b) $(x - 2)(4x + 6)$
(c) $(x - 2)(-4x - 6)$ (d) $(x - 2)(-4x + 6)$
(e) $(x - 2)(4x + 3)$

14. If x_1 and x_2 are the roots of $3x^2 - 2x - 6 = 0$, then $x_1^2 + x_2^2$ is equal to

(a) $\frac{50}{9}$ (b) $\frac{40}{9}$ (c) $\frac{30}{9}$ (d) $\frac{20}{9}$
(e) $\frac{10}{9}$

15. Let x_1 and x_2 be the roots of the equation $x^2 + px - 3 = 0$. If $x_1^2 + x_2^2 = 10$, then the value of p is equal to

(a) −4 or 4 (b) −3 or 3
(c) −2 or 2 (d) −1 or 1
(e) 0

16. If the product of roots of the equation $mx^2 + 6x + (2m - 1) = 0$ is −1, then the value of m is

(a) $\frac{1}{3}$ (b) 1 (c) 3 (d) −1
(e) −3

17. If $f(x)=\frac{1}{x^2+4x+4}-\frac{4}{x^4+4x^3+4x^2}+\frac{4}{x^3+2x^2}$,

then $f\left(\frac{1}{2}\right)$ is equal to

(a) 1 (b) 2 (c) −1 (d) 3
(e) 4

18. If x and y are the roots of the equation $x^2 + bx + 1 = 0$, then the value of $\frac{1}{x+b}+\frac{1}{y+b}$ is

(a) $\frac{1}{b}$ (b) b (c) $\frac{1}{2b}$ (d) $2b$
(e) 1

19. The equations $x^5 + ax + 1 = 0$ and $x^6 + ax^2 + 1 = 0$ have a common root. Then a is equal to

(a) −4 (b) −2 (c) −3 (d) −1
(e) 0

20. The roots of $ax^2 + x + 1 = 0$, where $a \neq 0$, are in the ratio 1 : 1. Then a is equal to

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{3}{4}$ (d) 1
(e) 0

21. If $z^2 + z + 1 = 0$ where z is a complex number, then the value of $\left(z+\frac{1}{z}\right)^2+\left(z^2+\frac{1}{z^2}\right)^2+\left(z^3+\frac{1}{z^3}\right)^2$ equals

(a) 4 (b) 5 (c) 6 (d) 7
(e) 8

22. Let $\Delta=\begin{vmatrix}1 & 1 & 1\\ 1 & -1-w^2 & w^2\\ 1 & w & w^4\end{vmatrix}$, where $w \neq 1$ is a complex number such that $w^3 = 1$. Then Δ equals

(a) $3w + w^2$ (b) $3w^2$
(c) $3(w - w^2)$ (d) $-3w^2$
(e) $3w^2 + 1$

23. If $\begin{vmatrix}3i & -9i & 1\\ 2 & 9i & -1\\ 10 & 9 & i\end{vmatrix} = x + iy$, then

(a) $x = 1, y = 1$ (b) $x = 0, y = 1$
(c) $x = 1, y = 0$ (d) $x = 0, y = 0$
(e) $x = -1, y = 0$

24. If $z=\cos\left(\frac{\pi}{3}\right)-i\sin\left(\frac{\pi}{3}\right)$, then $z^2 - z + 1$ is equal to

(a) 0 (b) 1 (c) −1 (d) $\frac{\pi}{2}$
(e) π

25. $\left(\frac{1+\cos\left(\frac{\pi}{12}\right)+i\sin\left(\frac{\pi}{12}\right)}{1+\cos\left(\frac{\pi}{12}\right)-i\sin\left(\frac{\pi}{12}\right)}\right)^{72}$ is equal to

(a) 0 (b) −1 (c) 1 (d) $\frac{1}{2}$
(e) $-\frac{1}{2}$

26. If $A=\begin{vmatrix}4 & k & k\\ 0 & k & k\\ 0 & 0 & k\end{vmatrix}$ and det $(A) = 256$, then $|k|$ equals

(a) 4 (b) 5 (c) 6 (d) 7
(e) 8

27. If $A=\begin{pmatrix}1 & 0\\ 1 & 1\end{pmatrix}$, then $A^n + nI$ is equal to

(a) I (b) nA
(c) $I + nA$ (d) $I - nA$
(e) $nA - I$

28. If $|z| = 5$ and $w=\frac{z-5}{z+5}$, then Re(w) is equal to

(a) 0 (b) $\frac{1}{25}$ (c) 25 (d) 1
(e) −1

29. If $A=\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}$, then A^{2017} is equal to

(a) $2^{2015}A$ (b) $2^{2016}A$
(c) $2^{2014}A$ (d) $2^{2017}A$
(e) $2^{2020}A$

30. If $a = e^{i\theta}$, then $\frac{1+a}{1-a}$ is equal to

(a) $\cot\frac{\theta}{2}$ (b) $\tan\theta$
(c) $i\cot\frac{\theta}{2}$ (d) $i\tan\frac{\theta}{2}$
(e) $2\tan\theta$

31. Three numbers x, y and z are in arithmetic progression. If $x + y + z = -3$ and $xyz = 8$, then $x^2 + y^2 + z^2$ is equal to

(a) 9 (b) 10 (c) 21 (d) 20
(e) 1

32. The 30th term of the arithmetic progression 10, 7, 4, ... is

(a) −90 (b) −87 (c) −77 (d) −67
(e) −57

33. The arithmetic mean of two numbers x and y is 3 and geometric mean is 1. Then $x^2 + y^2$ is equal to

(a) 30 (b) 31 (c) 32 (d) 33
(e) 34

34. The solution of $3^{2x-1} = 81^{1-x}$ is

(a) $\frac{2}{3}$ (b) $\frac{1}{6}$ (c) $\frac{7}{6}$ (d) $\frac{5}{6}$
(e) $\frac{1}{3}$

35. The sixth term in the sequence is 3, 1, $\frac{1}{3}$, ... is

(a) $\frac{1}{27}$ (b) $\frac{1}{9}$ (c) $\frac{1}{81}$ (d) $\frac{1}{17}$
(e) $\frac{1}{7}$

36. Three numbers are in arithmetic progression. Their sum is 21 and the product of the first number and the third number is 45. Then the product of these three numbers is

(a) 315 (b) 90 (c) 180 (d) 270
(e) 450

37. If a+1, $2a + 1$, $4a - 1$ are in arithmetic progression, then the value of a is

(a) 1 (b) 2 (c) 3 (d) 4
(e) 5

38. Two numbers x and y have arithmetic mean 9 and geometric mean 4. Then x and y are the roots of

(a) $x^2 -18x -16 = 0$ (b) $x^2 -18x +16 = 0$
(c) $x^2 +18x -16 = 0$ (d) $x^2 +18x +16 = 0$
(e) $x^2 -17x +16 = 0$

39. Three unbiased coins are tossed. The probability of getting at least 2 tails is

(a) $\frac{3}{4}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{3}$
(e) $\frac{2}{3}$

40. A single letter is selected from the word TRICKS. The probability that it is either T or R is

(a) $\frac{1}{36}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{2}{3}$
(e) $\frac{1}{3}$

41. From 4 red balls, 2 white balls and 4 black balls, four balls are selected. The probability of getting 2 red balls is

(a) $\frac{7}{21}$ (b) $\frac{8}{21}$ (c) $\frac{9}{21}$ (d) $\frac{10}{21}$
(e) $\frac{11}{21}$

42. In a class, 60% of the students know lesson I, 40% know lesson II and 20% know lesson I and lesson II. A student is selected at random. The probability that the student does not know lesson I and lesson II is

(a) 0 (b) $\frac{4}{5}$ (c) $\frac{3}{5}$ (d) $\frac{1}{5}$
(e) $\frac{2}{5}$

43. Two distinct numbers x and y are chosen from 1, 2, 3, 4, 5. The probability that the arithmetic mean of x and y is an integer is

(a) 0 (b) $\frac{1}{5}$ (c) $\frac{3}{5}$ (d) $\frac{2}{5}$
(e) $\frac{4}{5}$

44. The number of 3×3 matrices with entries -1 or $+1$ is
(a) 2^4 (b) 2^5 (c) 2^6 (d) 2^7
(e) 2^9

45. Let S be the set of all 2×2 symmetric matrices whose entries are either zero or one. A matrix X is chosen from S. The probability that the determinant of X is not zero is
(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{3}{4}$ (d) $\frac{1}{4}$
(e) $\frac{2}{9}$

46. The number of words that can be formed by using all the letters of the word PROBLEM only once is
(a) 5! (b) 6! (c) 7! (d) 8!
(e) 9!

47. The number of diagonals in a hexagon is
(a) 8 (b) 9 (c) 10 (d) 11
(e) 12

48. The sum of odd integers from 1 to 2001 is
(a) 1001^2 (b) 1000^2 (c) 1002^2 (d) 1003^2
(e) 999^2

49. Two balls are selected from two black and two red balls. The probability that the two balls will have no black ball is
(a) $\frac{1}{7}$ (b) $\frac{1}{5}$ (c) $\frac{1}{4}$ (d) $\frac{1}{3}$
(e) $\frac{1}{6}$

50. If $z = i^9 + i^{19}$, then z is equal to
(a) $0 + 0i$ (b) $1 + 0i$ (c) $0 + i$ (d) $1 + 2i$
(e) $1 + 3i$

51. The mean for the data 6, 7, 10, 12, 13, 4, 8, 12 is
(a) 9 (b) 8 (c) 7 (d) 6
(e) 5

52. The set of all real numbers satisfying the inequality $x - 2 < 1$ is
(a) $(3, \infty)$ (b) $[3, \infty)$
(c) $[-3, \infty)$ (d) $(-\infty, -3)$
(e) $(-\infty, 3)$

53. If $\frac{|x-3|}{x-3} > 0$, then
(a) $x \in (-3, \infty)$ (b) $x \in (3, \infty)$
(c) $x \in (2, \infty)$ (d) $x \in (1, \infty)$
(e) $x \in (-1, \infty)$

54. The mode of the data 8, 11, 9, 8, 11, 9, 7, 8, 7, 3, 2, 8 is
(a) 11 (b) 9 (c) 8 (d) 3
(e) 7

55. If the mean of six numbers is 41, then the sum of these numbers is
(a) 246 (b) 236 (c) 226 (d) 216
(e) 206

56. If $\int_0^x f(t)dt = x^2 + e^x (x > 0)$, then $f(1)$ is equal to
(a) $1 + e$ (b) $2 + e$ (c) $3 + e$ (d) e
(e) 0

57. $\int \frac{x+1}{x^{1/2}} dx =$
(a) $-x^{3/2} + x^{1/2} + c$ (b) $x^{1/2}$
(c) $\frac{2}{3}x^{3/2} + 2x^{1/2} + c$ (d) $x^{3/2} + x^{1/2} + c$
(e) $x^{3/2}$

58. In a flight 50 people speak Hindi, 20 speak English and 10 speak both English and Hindi. The number of people who speak at least one of the two languages is
(a) 40 (b) 50 (c) 20 (d) 80
(e) 60

59. If $f(x) = \frac{x+1}{x-1}$, then the value of $f(f(x))$ is equal to
(a) x (b) 0 (c) $-x$ (d) 1
(e) 2

60. Two dice are thrown simultaneously. What is the probability of getting two numbers whose product is even?
(a) $\frac{3}{4}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{2}{3}$
(e) $\frac{1}{16}$

61. $\lim_{x \to 0} \frac{\sqrt{2+x} - \sqrt{2-x}}{x}$ is equal to
(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}$
(c) 0 (d) Does not exist
(e) $\frac{1}{2\sqrt{2}}$

62. $\int \frac{dx}{e^x + e^{-x} + 2}$ is equal to

(a) $\frac{1}{e^x+1}+c$ (b) $\frac{-1}{e^x+1}+c$

(c) $\frac{1}{1+e^{-x}}+c$ (d) $\frac{1}{e^{-x}-1}+c$

(e) $\frac{1}{e^x-1}+c$

63. $\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)+\tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right)$ is equal to

(a) $\sec\theta$ (b) $2\sec\theta$ (c) $\sec\frac{\theta}{2}$ (d) $\sin\theta$

(e) $\cos\theta$

64. $\int_{-1}^{0} \frac{dx}{x^2+x+2}$ is equal to

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) π (d) 0

(e) $-\pi$

65. $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$ is equal to

(a) 0 (b) $-\pi$ (c) $\frac{3\pi}{2}$ (d) $\frac{\pi}{2}$

(e) $\frac{\pi}{4}$

66. If (x, y) is equidistant from $(a + b, b - a)$ and $(a - b, a + b)$, then

(a) $x + y = 0$ (b) $bx - ay = 0$

(c) $ax - by = 0$ (d) $bx + ay = 0$

(e) $ax + by = 0$

67. If the points (1, 0), (0, 1) and $(x, 8)$ are collinear, then the value of x is equal to

(a) 5 (b) -6 (c) 6 (d) 7

(e) -7

68. The minimum value of the function $\max\{x, x^2\}$ is equal to

(a) 0 (b) 1 (c) 2 (d) $\frac{1}{2}$

(e) $\frac{3}{2}$

69. Let $f(x + y) = f(x) f(y)$ for all x and y. If $f(0) = 1$, $f(3) = 3$ and $f'(0) = 11$, then $f'(3)$ is equal to

(a) 11 (b) 22 (c) 33 (d) 44

(e) 55

70. If $f(9) = f'(9) = 0$, then $\lim_{x\to 9} \frac{\sqrt{f(x)}-3}{\sqrt{x}-3}$ is equal to

(a) 0 (b) $f(0)$ (c) $f'(3)$ (d) $f(9)$

(e) 1

71. The value of $\cos\left(\frac{\pi}{4}+x\right)+\cos\left(\frac{\pi}{4}-x\right)$ is

(a) $\sqrt{2}\sin^2 x$ (b) $\sqrt{2}\sin x$

(c) $\sqrt{2}\cos^2 x$ (d) $\sqrt{3}\cos x$

(e) $\sqrt{2}\cos x$

72. Area of the triangle with vertices (–2, 2), (1, 5) and (6, –1) is

(a) 15 (b) $\frac{3}{5}$ (c) $\frac{29}{2}$ (d) $\frac{33}{2}$

(e) $\frac{35}{2}$

73. The equation of the line passing through(–3, 5) and perpendicular to the line through the points (1, 0) and (–4, 1) is

(a) $5x + y + 10 = 0$ (b) $5x - y + 20 = 0$

(c) $5x - y - 10 = 0$ (d) $5x + y + 20 = 0$

(e) $5y - x - 10 = 0$

74. The coefficient of x^5 in the expansion of $(1 + x^2)^5 (1 + x)^4$ is

(a) 30 (b) 60 (c) 40 (d) 10

(e) 45

75. The coefficient of x^4 in the expansion of $(1 - 2x)^5$ is equal to

(a) 40 (b) 320 (c) -320 (d) -32

(e) 80

76. The equation $5x^2 + y^2 + y = 8$ represents

(a) an ellipse (b) a parabola

(c) a hyperbola (d) a circle

(e) a straight line

77. The centre of the ellipse $4x^2 + y^2 - 8x + 4y - 8 = 0$ is

(a) (0, 2) (b) (2, –1) (c) (2, 1) (d) (1, 2)

(e) (1, –2)

78. The area bounded by the curves $y = -x^2 + 3$ and $y = 0$ is

(a) $\sqrt{3}+1$ (b) $\sqrt{3}$

(c) $4\sqrt{3}$ (d) $5\sqrt{3}$

(e) $6\sqrt{3}$

79. The order of the differential equation

$$\left(\frac{d^3y}{dx^3}\right)^2+\left(\frac{d^2y}{dx^2}\right)^2+\left(\frac{dy}{dx}\right)^5 = 0 \text{ is}$$

(a) 3 (b) 4 (c) 1 (d) 5
(e) 6

80. If $f(x) = \sqrt{2x} + \frac{4}{\sqrt{2x}}$, then $f'(2)$ is equal to
(a) 0 (b) −1 (c) 1 (d) 2
(e) −2

81. The area of the circle $x^2 - 2x + y^2 - 10y + k = 0$ is 25π. The value of k is equal to
(a) −1 (b) 1 (c) 0 (d) 2
(e) 3

82. $\int_{2016}^{2017} \frac{\sqrt{x}}{\sqrt{x}+\sqrt{4033-x}}dx$ is equal to
(a) $\frac{1}{4}$ (b) $\frac{3}{2}$ (c) $\frac{2017}{2}$ (d) $\frac{1}{2}$
(e) 508

83. The solution of $\frac{dy}{dx} + y \tan x = \sec x$, $y(0) = 0$ is
(a) $y \sec x = \tan x$ (b) $y \tan x = \sec x$
(c) $\tan x = y \tan x$ (d) $x \sec x = \tan y$
(e) $y \cot x = \sec x$

84. If the vectors $2\hat{i}+2\hat{j}+6\hat{k}, 2\hat{i}+\lambda\hat{j}+6\hat{k}, 2\hat{i}-3\hat{j}+\hat{k}$ are coplanar, then the value of λ is
(a) −10 (b) 1 (c) 0 (d) 10
(e) 2

85. The distance between (2, 1, 0) and $2x + y + 2z + 5 = 0$ is
(a) 10 (b) $\frac{10}{3}$ (c) $\frac{10}{9}$ (d) 5
(e) 1

86. The equation of the hyperbola with vertices (0, ± 15) and foci (0, ± 20) is
(a) $\frac{x^2}{175} - \frac{y^2}{225} = 1$ (b) $\frac{x^2}{625} - \frac{y^2}{125} = 1$
(c) $\frac{y^2}{225} - \frac{x^2}{125} = 1$ (d) $\frac{y^2}{65} - \frac{x^2}{65} = 1$
(e) $\frac{y^2}{225} - \frac{x^2}{175} = 1$

87. The value of $\frac{15^3+6^3+3\cdot 6\cdot 15\cdot 21}{1+4(6)+6(36)+4(216)+1296}$ is equal to
(a) $\frac{29}{7}$ (b) $\frac{7}{19}$ (c) $\frac{6}{17}$ (d) $\frac{21}{19}$
(e) $\frac{27}{7}$

88. The equation of the plane that passes through the points (1, 0, 2), (−1, 1, 2), (5, 0, 3) is
(a) $x + 2y - 4z + 7 = 0$ (b) $x + 2y - 3z + 7 = 0$
(c) $x - 2y + 4z + 7 = 0$ (d) $2y - 4z - 7 + x = 0$
(e) $x + 2y + 3z + 7 = 0$

89. The vertex of the parabola $y^2 - 4y - x + 3 = 0$ is
(a) (−1, 3) (b) (−1, 2)
(c) (2, −1) (d) (3, −1)
(e) (1, 2)

90. If $\vec{a}, \vec{b}, \vec{c}$ are vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ and $|\vec{a}|=7, |\vec{b}|=5, |\vec{c}|=3$, then the angle between $\vec{c}$ and $\vec{b}$ is
(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{4}$ (d) π
(e) 0

91. Let $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$, where $a > 0$. The minimum of f is attained at a point q and the maximum is attained at a point p. If $p^3 = q$, then a is equal to
(a) 1 (b) 3 (c) 2 (d) $\sqrt{2}$
(e) $\frac{1}{2}$

92. For all real numbers x and y, it is known that the real valued function f satisfies $f(x) + f(y) = f(x + y)$. If $f(1) = 7$, then $\sum_{r=1}^{100} f(r)$ is equal to
(a) $7 \times 51 \times 102$ (b) $6 \times 50 \times 102$
(c) $7 \times 50 \times 102$ (d) $6 \times 25 \times 102$
(e) $7 \times 50 \times 101$

93. The eccentricity of the ellipse
$\frac{(x-1)^2}{2} + \left(y+\frac{3}{4}\right)^2 = \frac{1}{16}$ is
(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2\sqrt{2}}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$
(e) $\frac{1}{4\sqrt{2}}$

94. $\int_{-1}^{1} \max\{x, x^3\}dx$ is equal to
(a) $\frac{3}{4}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) 1
(e) 0

95. If $x \in \left[0, \frac{\pi}{2}\right], y \in \left[0, \frac{\pi}{2}\right]$ and $\sin x + \cos y = 2$, then the value of $x + y$ is equal to

(a) 2π (b) π (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$
(e) 0

96. Let a, $a + r$ and $a + 2r$ be positive real numbers such that their product is 64. Then the minimum value of $a + 2r$ is equal to
(a) 4 (b) 3 (c) 2 (d) $\frac{1}{2}$
(e) 1

97. The sum $S=\frac{1}{9!}+\frac{1}{3!7!}+\frac{1}{5!5!}+\frac{1}{7!3!}+\frac{1}{9!}$ is equal to
(a) $\frac{2^{10}}{8!}$ (b) $\frac{2^9}{10!}$ (c) $\frac{2^7}{10!}$ (d) $\frac{2^6}{10!}$
(e) $\frac{2^5}{8!}$

98. If $f(x)=\begin{vmatrix} x & x^2 & x^3 \\ 1 & 2x & 3x^2 \\ 0 & 2 & 6x \end{vmatrix}$, then $f'(x)$ is equal to
(a) $x^3 + 6x^2$ (b) $6x^3$
(c) $3x$ (d) $6x^2$
(e) 0

99. $\int \frac{x^2}{1+(x^3)^2}dx$ is equal to
(a) $\tan^{-1}(x^2) + c$ (b) $\frac{2}{3}\tan^{-1}(x^3)+c$
(c) $\frac{1}{3}\tan^{-1}(x^3)+c$ (d) $\frac{1}{2}\tan^{-1}(x^2)+c$
(e) $\tan^{-1}(x^3) + c$

100. Let $f_n(x)$ be the n^{th} derivative of $f(x)$. The least value of n so that $f_n = f_{n+1}$, where $f(x) = x^2 + e^x$ is
(a) 4 (b) 5 (c) 2 (d) 3
(e) 6

101. $\sin 765°$ is equal to
(a) 1 (b) 0 (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{2}$
(e) $\frac{1}{\sqrt{2}}$

102. The distance of the point (3, –5) from the line $3x – 4y – 26 = 0$ is
(a) $\frac{3}{7}$ (b) $\frac{2}{5}$ (c) $\frac{7}{5}$ (d) $\frac{3}{5}$
(e) 1

103. The difference between the maximum and minimum value of the function
$f(x)=\int_0^x (t^2+t+1)dt$ on [2, 3] is
(a) $\frac{39}{6}$ (b) $\frac{49}{6}$ (c) $\frac{59}{6}$ (d) $\frac{69}{6}$
(e) $\frac{79}{6}$

104. If a and b are the non zero distinct roots of $x^2 + ax + b = 0$, then the minimum value of $x^2 + ax + b$ is
(a) $\frac{2}{3}$ (b) $\frac{9}{4}$ (c) $\frac{-9}{4}$ (d) $\frac{-2}{3}$
(e) 1

105. If the straight line $y = 4x + c$ touches the ellipse $\frac{x^2}{4}+y^2=1$ then c is equal to
(a) 0 (b) $\pm\sqrt{65}$ (c) $\pm\sqrt{62}$ (d) $\pm\sqrt{2}$
(e) $\pm$ 13

106. The equations $\lambda x – y = 2$, $2x – 3y = – \lambda$ and $3x – 2y = –1$ are consistent for
(a) $\lambda = – 4$ (b) $\lambda = 1, 4$
(c) $\lambda = 1, – 4$ (d) $\lambda = –1, 4$
(e) $\lambda = –1$

107. The set $\{(x, y):|x|+|y|=1\}$ in the xy plane represents
(a) a square (b) a circle
(c) an ellipse
(d) a rectangle which is not a square
(e) a rhombus which is not a square

108. The value of $\cos\left(\tan^{-1}\left(\frac{3}{4}\right)\right)$ is
(a) $\frac{4}{5}$ (b) $\frac{3}{5}$ (c) $\frac{3}{4}$ (d) $\frac{2}{5}$
(e) 0

109. Let $A(6, –1)$, $B(1, 3)$ and $C(x, 8)$ be three points such that $AB = BC$. The values of x are
(a) 3, 5 (b) –3, 5 (c) 3, –5 (d) 4, 5
(e) –3, –5

110. In an experiment with 15 observations on x, the following results were available $\sum x^2 = 2830$ and $\sum x = 170$. One observation that was 20, was found to be wrong and was replaced by the correct value 30. Then the corrected variance is
(a) 9.3 (b) 8.3 (c) 188.6 (d) 177.3
(e) 78

111. The angle between the pair of lines $\frac{x-2}{2}=\frac{y-1}{5}=\frac{z+3}{-3}$ and $\frac{x+2}{-1}=\frac{y-4}{8}=\frac{z-5}{4}$ is

(a) $\cos^{-1}\left(\frac{21}{9\sqrt{38}}\right)$ (b) $\cos^{-1}\left(\frac{23}{9\sqrt{38}}\right)$

(c) $\cos^{-1}\left(\frac{24}{9\sqrt{38}}\right)$ (d) $\cos^{-1}\left(\frac{25}{9\sqrt{38}}\right)$

(e) $\cos^{-1}\left(\frac{26}{9\sqrt{38}}\right)$

112. Let $\vec{a}$ be a unit vector. If $(\vec{x}-\vec{a})\cdot(\vec{x}+\vec{a})=12,$ then the magnitude of $\vec{x}$ is

(a) $\sqrt{8}$ (b) $\sqrt{9}$ (c) $\sqrt{10}$ (d) $\sqrt{13}$

(e) $\sqrt{12}$

113. The area of the triangular region whose sides are $y = 2x + 1$, $y = 3x + 1$ and $x = 4$ is

(a) 5 (b) 6 (c) 7 (d) 8

(e) 9

114. If ${}^nC_{r-1} = 36$, ${}^nC_r = 84$ and ${}^nC_{r+1} = 126$, then the value of r is

(a) 9 (b) 3 (c) 4 (d) 5

(e) 6

115. Let $f(x + y) = f(x)\,f(y)$ and $f(x) = 1 + \sin(3x)g(x)$, where g is differentiable. Then $f'(x)$ is equal to

(a) $3f(x)$ (b) $g(0)$

(c) $f(x)g(0)$ (d) $3g(x)$

(e) $3f(x)g(0)$

116. The roots of the equation $\begin{vmatrix} x-1 & 1 & 1 \\ 1 & x-1 & 1 \\ 1 & 1 & x-1 \end{vmatrix}=0$ are

(a) 1, 2 (b) –1, 2 (c) –1, –2 (d) 1, –2

(e) 1, 1

117. If the 7th and 8th term of the binomial expansion $(2a-3b)^n$ are equal, then $\frac{2a+3b}{2a-3b}$ is equal to

(a) $\frac{13-n}{n+1}$ (b) $\frac{n+1}{13-n}$

(c) $\frac{6-n}{13-n}$ (d) $\frac{n-1}{13-n}$

(e) $\frac{2n-1}{13-n}$

118. Standard deviation of first n odd natural numbers is

(a) $\sqrt{n}$ (b) $\sqrt{\frac{(n+2)(n+1)}{3}}$

(c) $\sqrt{\frac{n^2-1}{3}}$ (d) n

(e) $2n$

119. Let $S = \{1, 2, 3, ..., 10\}$. The number of subsets of S containing only odd numbers is

(a) 15 (b) 31 (c) 63 (d) 7

(e) 5

120. The area of the parallelogram with vertices (0, 0), (7, 2) (5, 9) and (12, 11) is

(a) 50 (b) 54 (c) 51 (d) 52

(e) 53

SOLUTIONS

1. **(a) :** We have,

$$\begin{vmatrix} 1 & 1 & 1 \\ p & q & r \\ p & q & r+1 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ p & q & r \\ p & q & r \end{vmatrix} + \begin{vmatrix} 1 & 1 & 0 \\ p & q & 0 \\ p & q & 1 \end{vmatrix}$$

$$= 0 + \begin{vmatrix} 1 & 1 & 0 \\ p & q & 0 \\ p & q & 1 \end{vmatrix} \quad (\because\ R_2 \sim R_3 \text{ in } 1^{st} \text{ determinant})$$

$$= \begin{vmatrix} 1 & 1 & 0 \\ p & q & 0 \\ p & q & 1 \end{vmatrix} = 1(q-p)$$

2. **(b) :** Given, $A = \begin{bmatrix} 5 & 0 \\ 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

Now, $4A + 5B - C = O$

$$\Rightarrow\ C = 4A + 5B = 4\begin{bmatrix} 5 & 0 \\ 1 & 0 \end{bmatrix} + 5\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 20 & 0 \\ 4 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 5 \\ -5 & 0 \end{bmatrix} = \begin{bmatrix} 20 & 5 \\ -1 & 0 \end{bmatrix}$$

3. **(a) :** Given, $U = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$

Since, $U^{-1} = \frac{1}{|U|}\,\text{adj}(U)$

Hence, adj $(U) = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}^T = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$

$$|U| = \left(\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}\right) - \left(\frac{-1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}\right) = \frac{1}{2} + \frac{1}{2} = 1$$

$\therefore \quad U^{-1} = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} = U^T$

4. (a) : We have, $A = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

Hence, $\text{adj}(A) = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}^T = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Now, $|A| = 0 + (1)(-1) = -1$

$\therefore \quad A^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} = A^T$

5. (a) : We have, $\begin{pmatrix} x+y & x-y \\ 2x+z & x+z \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}$

On comparing, we get

$x + y = 0$ ...(i), $x - y = 0$ ...(ii)

$2x + z = 1$ ...(iii), $x + z = 1$ ...(iv)

On solving (iii) and (iv), we get $x = 0$, $z = 1$.

Hence, from (i), we get $y = 0$

$\therefore \quad x = 0, y = 0, z = 1.$

6. (b) : We have, $\begin{pmatrix} 7 & 1 & 5 \\ 8 & 0 & 0 \end{pmatrix}\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} + 5\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$$= \begin{pmatrix} 14+3+5 \\ 16+0+0 \end{pmatrix} + \begin{pmatrix} 5 \\ 0 \end{pmatrix} = \begin{pmatrix} 22 \\ 16 \end{pmatrix} + \begin{pmatrix} 5 \\ 0 \end{pmatrix} = \begin{pmatrix} 27 \\ 16 \end{pmatrix}$$

7. (d) : Let $A = \begin{pmatrix} 1 & 2 & 4 \\ 1 & 3 & 5 \\ 1 & 4 & a \end{pmatrix}$

Since A is singular matrix $\quad \therefore \quad |A| = 0$

Hence, $\begin{vmatrix} 1 & 2 & 4 \\ 1 & 3 & 5 \\ 1 & 4 & a \end{vmatrix} = 0$

$\Rightarrow \quad 1(3a - 20) - 2(a - 5) + 4(4 - 3) = 0 \Rightarrow a = 6$

8. (d) : We have, $\begin{pmatrix} 1 & 2 & -3 \\ 0 & 4 & 5 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

$$\Rightarrow \quad \begin{pmatrix} x+2y-3z \\ 4y+5z \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

On comparing, we get

$x + 2y - 3z = 1$ ...(i) $\quad 4y + 5z = 1$ ...(ii)

and $z = 1$ ...(iii)

On solving (i), (ii) and (iii), we get $z = 1$, $y = -1$ and $x = 6$

9. (b) : We have, $A = \begin{pmatrix} 1 & 5 \\ 0 & 2 \end{pmatrix}$

$$A^2 = A \cdot A = \begin{pmatrix} 1 & 5 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 1 & 5 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 15 \\ 0 & 4 \end{pmatrix}$$

$$3A = 3\begin{pmatrix} 1 & 5 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 3 & 15 \\ 0 & 6 \end{pmatrix}$$

$$2I = 2\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

$$\therefore \quad A^2 - 3A + 2I = \begin{pmatrix} 1 & 15 \\ 0 & 4 \end{pmatrix} - \begin{pmatrix} 3 & 15 \\ 0 & 6 \end{pmatrix} + \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = O$$

10. (a) : We have, $\begin{pmatrix} 2x+y & x+y \\ p-q & p+q \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$

On comparing, we get

$2x + y = 1$ ...(i), $\quad x + y = 1$ ...(ii)

$p - q = 0$ ...(iii), $\quad p + q = 0$ ...(iv)

On solving (i) and (ii), we get $x = 0$ and $y = 1$

And, on solving (iii) and (iv), we get $p = 0$ and $q = 0$

$\therefore \quad (x, y, p, q) = (0, 1, 0, 0)$

11. (b) : Let $\left|\sqrt{4+2\sqrt{3}}\right| - \left|\sqrt{4-2\sqrt{3}}\right| = x$

On squaring both sides, we get

$$\left(\sqrt{4+2\sqrt{3}} - \sqrt{4-2\sqrt{3}}\right)^2 = x^2$$

$$\Rightarrow \quad (4+2\sqrt{3}) + (4-2\sqrt{3}) - 2\left(\sqrt{4+2\sqrt{3}}\right)\left(\sqrt{4-2\sqrt{3}}\right) = x^2$$

$$\Rightarrow \quad 8 - 2\sqrt{16-12} = x^2 \quad \Rightarrow \quad 8 - 4 = x^2$$

$$\Rightarrow \quad x^2 = 4 \quad \Rightarrow \quad x = \pm 2$$

Hence, $\left|\sqrt{4+2\sqrt{3}}\right| - \left|\sqrt{4-2\sqrt{3}}\right| = \pm 2$

12. (a) : We have, $8^{2/3} - 16^{1/4} - 9^{1/2}$

$= ((2)^3)^{2/3} - ((2)^4)^{1/4} - ((3)^2)^{1/2}$

$= 4 - 2 - 3 = 4 - 5 = -1$

13. (a) : Given, $x = 2$ is a root of $y = 4x^2 - 14x + q = 0$ then, $x = 2$ satisfy the given equation *i.e.*, $y(2) = 0$

$\Rightarrow\ 4(2)^2 - 14(2) + q = 0$

$\Rightarrow\ 16 - 28 + q = 0 \Rightarrow q = 12$

$\therefore\ y = 4x^2 - 14x + 12 = 0$

$\Rightarrow\ y = 4x^2 - 6x - 8x + 12$

$\Rightarrow\ y = 2x(2x - 3) - 4(2x - 3)$

$\Rightarrow\ y = (2x - 4)(2x - 3) \Rightarrow y = (x - 2)(4x - 6)$

14. (b) : We have, $3x^2 - 2x - 6 = 0$

$\therefore$ Roots of given equation (x_1, x_2)

$$= \frac{-(-2) \pm \sqrt{4+72}}{6} = \frac{2 \pm \sqrt{76}}{6} = \frac{2 \pm 2\sqrt{19}}{6} = \frac{1 \pm \sqrt{19}}{3}$$

Hence, $x_1 = \dfrac{1+\sqrt{19}}{3}$ and $x_2 = \dfrac{1-\sqrt{19}}{3}$

Now, $x_1^2 + x_2^2 = \left(\dfrac{1+\sqrt{19}}{3}\right)^2 + \left(\dfrac{1-\sqrt{19}}{3}\right)^2$

$$= \frac{1}{9}(1 + 19 + 2\sqrt{19} + 1 + 19 - 2\sqrt{19}) = \frac{40}{9}$$

15. (c) : Given, x_1 and x_2 are roots of $x^2 + px - 3 = 0$. Also, $x_1^2 + x_2^2 = 10$

$\therefore$ Sum of roots $= (x_1 + x_2) = \dfrac{-p}{1} = -p$

And, product of roots $= x_1x_2 = \dfrac{-3}{1} = -3$

$\therefore\ (x_1 + x_2)^2 = p^2$

$\Rightarrow\ x_1^2 + x_2^2 + 2x_1x_2 = p^2 \Rightarrow 10 + 2(-3) = p^2$

$\Rightarrow\ p^2 = 4 \Rightarrow p = \pm 2$

Hence, the value of p is 2 or -2.

16. (a) : Given, $mx^2 + 6x + (2m - 1) = 0$ and product of roots is -1.

$$\Rightarrow \frac{(2m-1)}{m} = -1 \Rightarrow 2m - 1 = -m \Rightarrow 3m = 1 \Rightarrow m = \frac{1}{3}$$

17. (e) : We have,

$$f(x) = \frac{1}{x^2+4x+4} - \frac{4}{x^4+4x^3+4x^2} + \frac{4}{x^3+2x^2}$$

$$= \frac{1}{(x+2)^2} - \frac{4}{x^2(x+2)^2} + \frac{4}{x^2(x+2)}$$

$$\therefore f(1/2) = \frac{1}{\left(\frac{1}{2}+2\right)^2} - \frac{4}{\left(\frac{1}{2}\right)^2\left(\frac{1}{2}+2\right)^2} + \frac{4}{\left(\frac{1}{2}\right)^2\left(\frac{1}{2}+2\right)}$$

$$= \frac{1}{\left(\frac{5}{2}\right)^2} - \frac{4}{\frac{1}{4}\left(\frac{5}{2}\right)^2} + \frac{4}{\frac{1}{4}\left(\frac{5}{2}\right)}$$

$$= \frac{4}{25} - \frac{64}{25} + \frac{32}{5} = \frac{-60+160}{25} = \frac{100}{25} = 4$$

18. (b) : We have, $x^2 + bx + 1 = 0$ and x, y are its roots.

$\therefore$ Sum of roots $= (x + y) = -b$ ...(i)

And product of roots $= (xy) = 1$ ...(ii)

Now, $\dfrac{1}{x+b} + \dfrac{1}{y+b} = \dfrac{y+b+x+b}{(x+b)(y+b)}$

$$= \frac{(x+y)+2b}{xy+b(x+y)+b^2} = \frac{(-b)+2b}{1-b^2+b^2}$$

(Using (i) and (ii))

$= b$

19. (b) : Let y be the common root of

$x^5 + ax + 1 = 0$ and $x^6 + ax^2 + 1 = 0$

Then, $y^5 + ay + 1 = 0$ and $y^6 + ay^2 + 1 = 0$

$\Rightarrow\ y^5 + ay + 1 = y^6 + ay^2 + 1$

$\Rightarrow\ y^5 - y^6 + ay - ay^2 = 0$

$\Rightarrow\ y^5(1 - y) + ay(1 - y) = 0$

$\Rightarrow\ (y^5 + ay)(1 - y) = 0 \Rightarrow y = 1$

Hence, the common root is 1.

i.e., $1 + a + 1 = 0 \Rightarrow a = -2$

20. (a) : Let x_1 and x_2 be the roots of $ax^2 + x + 1 = 0$ then,

$x_1 + x_2 = \dfrac{-1}{a}$ ...(i) and $x_1x_2 = \dfrac{1}{a}$ ...(ii)

Also, $x_1 : x_2 = 1 : 1 \Rightarrow x_1 = x_2$ ...(iii)

Using (iii) in (i), we get $2x_1 = \dfrac{-1}{a} \Rightarrow x_1 = \dfrac{-1}{2a}$

Using (iii) in (ii), we get $x_1^2 = \dfrac{1}{a}$ $\therefore \dfrac{1}{4a^2} = \dfrac{1}{a}$

$\Rightarrow\ 4a = 1 \Rightarrow a = \dfrac{1}{4}$

21. (c) : We have, $z^2 + z + 1 = 0$

$\Rightarrow\ z = \omega$ or ω^2

where ω, ω^2 are complex cube roots of unity.

Now, $\left(z+\dfrac{1}{z}\right)^2 + \left(z^2+\dfrac{1}{z^2}\right)^2 + \left(z^3+\dfrac{1}{z^3}\right)^2$

$$= \left(\omega+\frac{1}{\omega}\right)^2 + \left(\omega^2+\frac{1}{\omega^2}\right)^2 + \left(\omega^3+\frac{1}{\omega^3}\right)^2$$

$$= \left(\frac{\omega^2+1}{\omega}\right)^2 + \left(\frac{\omega^4+1}{\omega^2}\right)^2 + (1+1)^2$$

[using $\omega^3 = 1$]

$$= \left(\frac{-\omega}{\omega}\right)^2 + \left(\frac{-\omega^2}{\omega^2}\right)^2 + 4 \quad [\text{using } 1+\omega+\omega^2 = 0]$$

$= 1 + 1 + 4 = 6$

22. (b) : We have, $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1-w^2 & w^2 \\ 1 & w & w^4 \end{vmatrix}$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1-w^2 & w^2 \\ 1 & w & w \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & w & w^2 \\ 1 & w & w \end{vmatrix}$$

$(\because \ 1 + w + w^2 = 0)$

Applying $C_1 \to C_1 - C_2$ and $C_2 \to C_2 - C_3$, we get

$$\Delta = \begin{vmatrix} 0 & 0 & 1 \\ 1-w & w-w^2 & w^2 \\ 1-w & 0 & w \end{vmatrix}$$

$= 1(0 - (1 - w)(w - w^2))$ (on expanding along R_1)

$= -(w - w^2 - w^2 + w^3)$

$= -(-1 - w^2 - w^2 - w^2 + 1) = -(-3w^2) = 3w^2$

23. (d) : We have, $\begin{vmatrix} 3i & -9i & 1 \\ 2 & 9i & -1 \\ 10 & 9 & i \end{vmatrix} = (x + iy)$

$\Rightarrow \ 3i(9i^2 + 9) + 9i(2i + 10) + (18 - 90i) = x + iy$

$\Rightarrow \ 3i(-9 + 9) + 18i^2 + 90i + 18 - 90i = x + iy$

$\Rightarrow \ -18 + 18 = x + iy$

$\Rightarrow \ 0 + 0i = x + iy$

On comparing, we get $x = 0$ and $y = 0$

24. (a) : We have, $z = \cos\left(\frac{\pi}{3}\right) - i\sin\left(\frac{\pi}{3}\right)$

$$= \frac{1}{2} - \frac{\sqrt{3}\,i}{2} = \frac{1-\sqrt{3}\,i}{2}$$

then, $\quad z^2 - z + 1 = \left(\frac{1-\sqrt{3}\,i}{2}\right)^2 - \left(\frac{1-\sqrt{3}\,i}{2}\right) + 1$

$$= \frac{1}{4}(1 - 3 - 2\sqrt{3}\,i) + \frac{1}{2} + \frac{\sqrt{3}\,i}{2}$$

$$= \frac{-1}{2} - \frac{\sqrt{3}i}{2} + \frac{1}{2} + \frac{\sqrt{3}i}{2} = 0$$

25. (c) : Let $z = \cos\left(\frac{\pi}{12}\right) + i\sin\left(\frac{\pi}{12}\right)$

$\therefore \ \frac{1}{z} = \left(\cos\frac{\pi}{12} - i\sin\frac{\pi}{12}\right)$

Now, from given expression, we have

$$\left(\frac{1+\cos\left(\frac{\pi}{12}\right)+i\sin\left(\frac{\pi}{12}\right)}{1+\cos\left(\frac{\pi}{12}\right)-i\sin\left(\frac{\pi}{12}\right)}\right)^{72} = \left(\frac{1+z}{1+z^{-1}}\right)^{72}$$

$$= \left(\frac{(1+z)z}{(z+1)}\right)^{72} = (z)^{72} = \left(\cos\left(\frac{\pi}{12}\right)+i\sin\left(\frac{\pi}{12}\right)\right)^{72}$$

$$= \cos\left(\frac{72\pi}{12}\right)+i\sin\left(\frac{72\pi}{12}\right)$$

(Using De-Moivre's theorem)

$= \cos 6\pi + i \sin 6\pi = 1$

26. (e) : We have, $A = \begin{vmatrix} 4 & k & k \\ 0 & k & k \\ 0 & 0 & k \end{vmatrix}$

Now, expanding along C_1, we get $|A| = 4(k^2)$

But $\det(A) = 256$ (Given)

$\therefore$ On comparing, we get

$4k^2 = 256 \ \Rightarrow \ k^2 = 64 \ \Rightarrow \ k = \pm 8$

Hence, $|k| = 8$

27. (c) : We have, $A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$

$$\therefore \quad A^2 = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$$

$$A^3 = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix} \quad \therefore \ A^n = \begin{pmatrix} 1 & 0 \\ n & 1 \end{pmatrix}$$

Hence, $A^n + nI = \begin{pmatrix} 1 & 0 \\ n & 1 \end{pmatrix} + n\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$$= \begin{pmatrix} 1 & 0 \\ n & 1 \end{pmatrix} + \begin{pmatrix} n & 0 \\ 0 & n \end{pmatrix} = \begin{pmatrix} 1+n & 0 \\ n & 1+n \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} n & 0 \\ n & n \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + n\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$$

$= I + nA$

28. (a) : Let $z = x + iy$

$\therefore \ |z| = 5 \ \Rightarrow \ \sqrt{x^2+y^2} = 5 \ \Rightarrow \ x^2 + y^2 = 25$

Now, $w = \frac{z-5}{z+5} = \frac{x+iy-5}{x+iy+5} = \frac{(x-5)+iy}{(x+5)+iy}$

On rationalizing the denominator, we get

$$\frac{((x-5)+iy)((x+5)-iy)}{(x+5)^2+(y)^2} = \frac{x^2-25+y^2+10yi}{(x+5)^2+y^2}$$

$$= \frac{10yi}{(x+5)^2+y^2} \qquad \text{[Using } x^2+y^2=25]$$

$$= 0 + \frac{10yi}{(x+5)^2+y^2} \qquad \therefore \quad R(w) = 0$$

29. (b) : We have, $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

$$\therefore \quad A^2 = A \cdot A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix}$$

$$= 2\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = 2A$$

$$A^3 = A^2 \cdot A = \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 4 & 4 \\ 4 & 4 \end{pmatrix}$$

$$= 4\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = 2^2 \cdot A$$

Similarly $\quad A^n = 2^{n-1} \cdot A$

$\therefore \quad A^{2017} = 2^{2017-1} \cdot A = 2^{2016} \cdot A$

30. (c) : We have, $a = e^{i\theta} = \cos\theta + i\sin\theta$ (polar form)

$$\therefore \quad \frac{1+a}{1-a} = \frac{1+\cos\theta+i\sin\theta}{1-(\cos\theta+i\sin\theta)}$$

$$= \frac{2\cos^2\frac{\theta}{2} + i\,2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2} - i\,2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$$

$$= \frac{2\cos\frac{\theta}{2}\left[\cos\frac{\theta}{2}+i\sin\frac{\theta}{2}\right]}{2\sin\frac{\theta}{2}\left[\sin\frac{\theta}{2}-i\cos\frac{\theta}{2}\right]}$$

$$= \cot\frac{\theta}{2}\left[\frac{\left(\cos\frac{\theta}{2}+i\sin\frac{\theta}{2}\right)\times\left(\sin\frac{\theta}{2}+i\cos\frac{\theta}{2}\right)}{\left(\sin\frac{\theta}{2}-i\cos\frac{\theta}{2}\right)\left(\sin\frac{\theta}{2}+i\cos\frac{\theta}{2}\right)}\right]$$

[Rationalizing the denominator]

$$= \cot\frac{\theta}{2}\left[\frac{\cos\frac{\theta}{2}\sin\frac{\theta}{2}+i\cos^2\frac{\theta}{2}+i\sin^2\frac{\theta}{2}-\cos\frac{\theta}{2}\sin\frac{\theta}{2}}{\sin^2\frac{\theta}{2}+\cos^2\frac{\theta}{2}}\right]$$

$$= i\cot\frac{\theta}{2}$$

31. (c) : We have, $x + y + z = -3$ ...(i)

On squaring both sides, we get

$(x + y + z)^2 = (-3)^2$

$\Rightarrow \quad x^2 + y^2 + z^2 + 2xy + 2yz + 2zx = 9$ ...(ii)

Also, x, y, z are in A.P. $\Rightarrow \quad 2y = x + z$

$\Rightarrow \quad 2y = -3 - y \;\Rightarrow\; y = -1$ ...(iii)

So, $xyz = 8 \quad \Rightarrow \quad xz = -8$ ...(iv)

Now, using (iii) and (iv) in (ii), we get

$x^2 + y^2 + z^2 - 2x - 2z - 16 = 9$

$\Rightarrow \quad x^2 + y^2 + z^2 - 2(x + z) - 16 = 9$

$\Rightarrow \quad x^2 + y^2 + z^2 + 4 - 16 = 9$

$\Rightarrow \quad x^2 + y^2 + z^2 = 9 + 12 = 21$

Hence, $x^2 + y^2 + z^2 = 21$

32. (c) : We have the series of an A.P. as 10, 7, 4....

$\therefore$ First term $(a) = 10$

Common difference $(d) = 7 - 10 = -3$

$\therefore \quad a_{30} = a + (30 - 1)d = 10 + 29(-3) = 10 - 87 = -77$

33. (e) : Given, arithmetic mean of x and y is 3

i.e., $\frac{x+y}{2} = 3 \quad \Rightarrow \quad x + y = 6$ ...(i)

and geometric mean of x and y is 1

i.e., $\sqrt{xy} = 1 \quad \Rightarrow \quad xy = 1$ ...(ii)

Squaring (i) on both sides, we get

$(x + y)^2 = (6)^2 \quad \Rightarrow \quad x^2 + y^2 + 2xy = 36$

$\Rightarrow \quad x^2 + y^2 + 2 = 36$ (Using (ii))

$\Rightarrow \quad x^2 + y^2 = 34$

34. (d) : We have, $3^{2x-1} = 81^{1-x}$

$\Rightarrow \quad (3)^{2x-1} = ((3)^4)^{1-x} \quad \Rightarrow \quad (3)^{2x-1} = (3)^{4-4x}$

$\therefore$ On comparing, we get, $2x - 1 = 4 - 4x$

$\Rightarrow \quad 6x = 5 \quad$ or $\quad x = \frac{5}{6}$

35. (c) : Given series 3, 1, $\frac{1}{3}$, ... forms a G.P.

where first term $(a) = 3$ and common ratio $(r) = \frac{1}{3}$

$\therefore$ Sixth term, $a_6 = ar^5$

$$= (3)\left(\frac{1}{3}\right)^5 = \left(\frac{1}{3}\right)^4 = \frac{1}{81}$$

36. (a) : Let three numbers in A.P. be a, b and c

Then, according to question

$a + b + c = 21$ ...(i) and $ac = 45$ ...(ii)

$\because \quad a, b$ and c are in A.P. $\quad \therefore \quad 2b = c + a$ ...(iii)

Substituting (iii) in (i), we get

$3b = 21 \Rightarrow b = 7$

Hence, product of these three numbers = abc

$= 7(45) = 315$ (Using (ii))

37. (b) : Given, $a + 1, 2a + 1, 4a - 1$ are in A.P.

$\therefore \quad 2(2a + 1) = 4a - 1 + a + 1$

$\Rightarrow \quad 4a + 2 = 5a \quad \Rightarrow \quad a = 2$

38. (b) : Given, arithmetic mean of x and y is 9

i.e., $\dfrac{x+y}{2} = 9 \Rightarrow x + y = 18$

Geometric mean of x and y is 4 *i.e.*, $\sqrt{xy} = 4 \Rightarrow xy = 16$

Now, sum of roots $(x + y) = 18$

Product of roots $(xy) = 16$

$\therefore$ Required quadratic equation is $x^2 - 18x + 16 = 0$

39. (c) : Total number of outcomes = 8 *i.e.*,

{HHH, HHT, HTH, THH, TTH, THT, HTT, TTT}

Number of favourable outcomes = 4

i.e., {TTH, THT, HTT, TTT} = 4

$\therefore \quad P(\text{getting atleast 2 tails}) = \dfrac{4}{8} = \dfrac{1}{2}$

40. (e) : Number of letters in TRICKS = 6

Number of favourable outcomes = {T, R} = 2

$\therefore \quad P(\text{either T or R}) = \dfrac{2}{6} = \dfrac{1}{3}$

41. (c) : Total number of outcomes = ${}^{10}C_4$

2 red balls can be selected from 4 red balls in ${}^4C_2$ ways

And, remaining 2 balls can be selected from 2 white balls and 4 black balls in ${}^6C_2$ ways.

$\therefore \quad \text{Required probability} = \dfrac{{}^4C_2 \times {}^6C_2}{{}^{10}C_4} = \dfrac{3}{7} \text{ or } \dfrac{9}{21}$

42. (d) : Let A and B be two events of knowing lesson I and lesson II respectively.

$\therefore$ According to question,

$P(A) = \dfrac{60}{100}$; $P(B) = \dfrac{40}{100}$

$P(A \cap B) = \dfrac{20}{100}$

$\therefore \quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \dfrac{60}{100} + \dfrac{40}{100} - \dfrac{20}{100} = \dfrac{80}{100}$

Hence, required probability = $P(A^C \cap B^C)$

$= P(A \cup B)^C = 1 - P(A \cup B)$

$= 1 - \dfrac{80}{100} = \dfrac{20}{100} = \dfrac{1}{5}$

43. (d) : Two distinct numbers can be chosen from 1, 2, 3, 4, 5 in ${}^5C_2$ ways.

Number of outcomes having arithmetic mean an integer *i.e.*, {(1, 3), (1, 5) (2, 4), (3, 5)} = 4

$\therefore \quad \text{Required probability} = \dfrac{4}{{}^5C_2} = \dfrac{4}{10} = \dfrac{2}{5}$

44. (e) : There are 9 elements in 3 × 3 matrices and each element can be filled in two ways either – 1 or 1.

$\therefore$ Total possible matrices = 2^9

45. (b) : Total possible set of 2 × 2 symmetric matrices of entries either zero or one = 8

Possbile set of matrices having determinant not zero = 4

$\therefore \quad \text{Required probability} = \dfrac{4}{8} = \dfrac{1}{2}$

46. (c) : Number of words that can be formed by using all 7 letters of word PROBLEM only once is 7!.

47. (b) : Number of diagonals in n-sided polygon

$= \dfrac{n(n-3)}{2}$

$\therefore \quad \text{Number of diagonals in hexagon} = \dfrac{6(6-3)}{2} = 9$

48. (a) : We know sum of n odd numbers = n^2

Number of odd terms from 1 to 2001 = 1001

$\therefore$ Sum of 1001 odd terms = $(1001)^2$

49. (e) : Total number of ways of selecting two balls = ${}^4C_2$

Number of ways of selecting 2 balls in which no ball is black = ${}^2C_2$

$\therefore \quad \text{Required probability} = \dfrac{{}^2C_2}{{}^4C_2} = \dfrac{1}{6}$

50. (a) : We have, $z = i^9 + i^{19} = (i^2)^4 \cdot i + (i^2)^9 \cdot i$

$= i + (-i) = 0 = 0 + 0i$

51. (a) : Given data is 6, 7, 10, 12, 13, 4, 8, 12

$\therefore \quad \text{Mean} = \dfrac{6+7+10+12+13+4+8+12}{8} = \dfrac{72}{8} = 9$

52. (e) : Given, $x - 2 < 1 \Rightarrow x < 1 + 2 = 3$

Hence, set of all real numbers satisfying the inequality $x - 2 < 1$ is $(-\infty, 3)$

53. (b) : Given, $\dfrac{|x-3|}{x-3} > 0$

$\Rightarrow \quad |x - 3| > 0 \Rightarrow x - 3 > 0 \Rightarrow x > 3$

$\therefore \quad x \in (3, \infty)$

54. (c) : From the given data 8 has highest frequency.

$\therefore$ Mode of the given data is 8.

55. (a) : Let the six numbers be $x_1, x_2, x_3, x_4, x_5, x_6$

Given, mean of six numbers = 41

$\therefore \quad \text{Mean} = \dfrac{x_1+x_2+x_3+x_4+x_5+x_6}{6} = 41$

$\Rightarrow \quad x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 246$

56. (b) : Given, $\int_0^x f(t)\,dt = x^2 + e^x \ (x > 0)$

$\Rightarrow \quad f(x) = 2x + e^x$

$\therefore \quad f(1) = 2(1) + e = 2 + e$

57. (c) : $\int \frac{x+1}{x^{1/2}}dx = \int \sqrt{x}\,dx + \int \frac{1}{\sqrt{x}}\,dx$

$= \frac{2}{3}x^{3/2} + 2x^{1/2} + c$

58. (e) : Let H and E be the two events of people speaking Hindi and English respectively.

$\therefore \quad n(H) = 50, \ n(E) = 20$

$n(H \cap E) = 10$

$\therefore \quad n(H \cup E) = n(H) + n(E) - n(H \cap E)$

$= 50 + 20 - 10 = 60$

i.e., number of people who speak atleast one of two languages is 60.

59. (a) : Given, $f(x) = \frac{x+1}{x-1}$

$\therefore \quad f(f(x)) = f\left(\frac{x+1}{x-1}\right) = \frac{\frac{x+1}{x-1}+1}{\frac{x+1}{x-1}-1}$

$= \frac{(x+1+x-1)}{(x+1-x+1)} = \frac{2x}{2} = x$

60. (a) : Total number of outcomes = $6 \times 6 = 36$

Favourable number of outcomes = 27

$\therefore \quad$ Required probability $= \frac{27}{36} = \frac{3}{4}$

61. (a) : We have, $\lim_{x\to 0} \frac{\sqrt{2+x}-\sqrt{2-x}}{x}$

On rationalizing the numerator, we get

$\lim_{x\to 0} \frac{(2+x)-(2-x)}{x(\sqrt{2+x}+\sqrt{2-x})} = \lim_{x\to 0} \frac{2x}{x(\sqrt{2+x}+\sqrt{2-x})}$

$= \lim_{x\to 0} \frac{2}{\sqrt{2+x}+\sqrt{2-x}} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$

62. (b) : We have, $\int \frac{dx}{e^x + e^{-x} + 2} = \int \frac{e^x dx}{e^{2x}+1+2e^x}$

$= \int \frac{e^x dx}{(e^x+1)^2} = \int \frac{dt}{t^2}$

[Put $(e^x + 1) = t \Rightarrow e^x dx = dt$]

$= \frac{-1}{t} + c = \frac{-1}{e^x+1} + c$

63. (b) : We have, $\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right) + \tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right)$

$= \frac{\tan\frac{\pi}{4}+\tan\frac{\theta}{2}}{1-\tan\frac{\pi}{4}\tan\frac{\theta}{2}} + \frac{\tan\frac{\pi}{4}-\tan\frac{\theta}{2}}{1+\tan\frac{\pi}{4}\tan\frac{\theta}{2}}$

$= \frac{1+\tan\frac{\theta}{2}}{1-\tan\frac{\theta}{2}} + \frac{1-\tan\frac{\theta}{2}}{1+\tan\frac{\theta}{2}}$

$= \frac{\left(1+\tan\frac{\theta}{2}\right)^2 + \left(1-\tan\frac{\theta}{2}\right)^2}{\left(1-\tan^2\frac{\theta}{2}\right)} = \frac{2\left(1+\tan^2\frac{\theta}{2}\right)}{1-\tan^2\frac{\theta}{2}}$

$= 2\left(\frac{1}{\cos\theta}\right) = 2\sec\theta$

64. (None of the options is correct) :

We have, $\int_{-1}^{0} \frac{dx}{x^2+x+2}$

$= \int_{-1}^{0} \frac{dx}{x^2+x+2+\frac{1}{4}-\frac{1}{4}} = \int_{-1}^{0} \frac{dx}{\left(x+\frac{1}{2}\right)^2+\frac{7}{4}}$

$= \int_{-1}^{0} \frac{dx}{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{7}}{2}\right)^2} = \frac{1}{\sqrt{7}/2}\tan^{-1}\frac{x+1/2}{\sqrt{7}/2}\Bigg|_{-1}^{0}$

$= \frac{2}{\sqrt{7}}\left[\tan^{-1}\left(\frac{1/2}{\sqrt{7}/2}\right) - \tan^{-1}\left(\frac{-1/2}{\sqrt{7}/2}\right)\right]$

$= \frac{2}{\sqrt{7}}\left(\tan^{-1}\frac{1}{\sqrt{7}} + \tan^{-1}\frac{1}{\sqrt{7}}\right) = \frac{4}{\sqrt{7}}\tan^{-1}\left(\frac{1}{\sqrt{7}}\right)$

65. (e) : Let $I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$ ...(i)

$\Rightarrow \quad I = \int_0^{\pi/2} \frac{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}\,dx}{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}+\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}$

$\Rightarrow \quad I = \int_0^{\pi/2} \frac{\sqrt{\cos x}\,dx}{\sqrt{\cos x}+\sqrt{\sin x}}$ ...(ii)

Adding (i) and (ii), we get

$$2I = \int_0^{\pi/2} \left(\frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}\right) dx$$

$$\Rightarrow \quad I = \frac{1}{2}\int_0^{\pi/2} 1\cdot dx = \frac{1}{2}\left[\frac{\pi}{2}\right] = \frac{\pi}{4}$$

66. (b) : Given, (x, y) is equidistant from $(a + b, b - a)$ and $(a - b, a + b)$

$\therefore$ Using distance formula, we have

$$\sqrt{(x-(a+b))^2+(y-(b-a))^2} = \sqrt{(x-(a-b))^2+(y-(a+b))^2}$$

On squaring both sides, we get

$$(x - (a + b))^2 + (y - (b - a))^2 = (x - (a - b))^2 + (y - (a + b))^2$$

$$\Rightarrow x^2 + a^2 + b^2 + 2ab - 2ax - 2bx + y^2 + b^2 + a^2 - 2ab - 2by + 2ay$$
$$= x^2 + a^2 + b^2 - 2ab + 2bx - 2ax + y^2 + a^2 + b^2 + 2ab - 2by - 2ay$$

$$\Rightarrow -4bx + 4ay = 0 \quad \Rightarrow \quad ay - bx = 0 \text{ or } bx - ay = 0$$

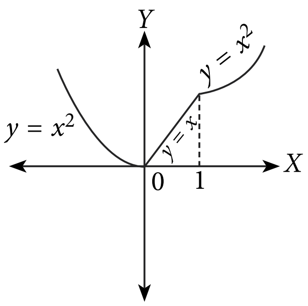
67. (e) : Given, (1, 0), (0, 1) and $(x, 8)$ are collinear

$\therefore$ Area of Δ formed by these points is zero

i.e., $\frac{1}{2}|1(1 - 8) + 0(8 - 0) + x(0 - 1)| = 0$

$$\Rightarrow \frac{1}{2}|-7 - x| = 0 \quad \Rightarrow \quad x = -7$$

68. (a) : Graph of $\max\{x, x^2\}$ is shown below



Hence, min value of $\max\{x, x^2\}$ is 0.

69. (c) : Given, $f(x + y) = f(x) f(y)$

$$f'(3) = \lim_{h\to 0}\frac{f(3+h)-f(3)}{h}$$

$$= \lim_{h\to 0}\frac{f(3)f(h)-f(3)}{h} = f(3)\lim_{h\to 0}\frac{f(h)-1}{h}$$

$$= f(3)\lim_{h\to 0}\frac{f(h)-f(0)}{h} \qquad [\because f(0) = 1]$$

$$= f(3)\lim_{h\to 0}\frac{f(0+h)-f(0)}{h} = f(3)\, f'(0)$$

$= 3 \times 11 = 33 \quad \therefore \quad f'(3) = 33$

70. (a) : We have, $f'(9) = \lim\limits_{x\to 9}\frac{f(x)-f(9)}{x-9}$

$$\Rightarrow \quad 0 = \lim_{x\to 9}\frac{f(x)}{x-9}\times\frac{\sqrt{f(x)}-3}{\sqrt{f(x)}-3}$$

$$\Rightarrow \quad 0 = \lim_{x\to 9}\left\{\frac{\sqrt{f(x)}-3}{(\sqrt{x}-3)(\sqrt{x}+3)}\times\frac{f(x)+9-9}{\sqrt{f(x)}-3}\right\}$$

$$\Rightarrow \quad 0 = \lim_{x\to 9}\left(\frac{\sqrt{f(x)}-3}{(\sqrt{x}-3)}\right)\times\left[\lim_{x\to 9}\left\{\frac{\left(\sqrt{f(x)}\right)^2-(3)^2}{\left(\sqrt{f(x)}-3(\sqrt{x}+3\right)}\right\} - \lim_{x\to 9}\left(\frac{9}{(\sqrt{f(x)}-3)\,(\sqrt{x}+3)}\right)\right]$$

$$\Rightarrow \quad 0 = \lim_{x\to 0}\left(\frac{\sqrt{f(x)}+3}{\sqrt{x}-3}\right)\times\left[\lim_{x\to 9}\frac{\sqrt{f(x)}+3}{\sqrt{x}+3}-\frac{9}{(\sqrt{f(9)}-3)(\sqrt{9}+3)}\right]$$

$$\Rightarrow \quad 0 = \lim_{x\to 9}\left(\frac{\sqrt{f(x)}+3}{\sqrt{x}-3}\right)\times\left[\frac{3}{6}+\frac{9}{3\times 6}\right]$$

$$\Rightarrow \quad \lim_{x\to 9}\left(\frac{\sqrt{f(x)}+3}{\sqrt{x}-3}\right) = 0$$

71. (e) : $\cos\left(\frac{\pi}{4}+x\right) + \cos\left(\frac{\pi}{4}-x\right)$

$$= \cos\frac{\pi}{4}\cos x - \sin\frac{\pi}{4}\sin x + \cos\frac{\pi}{4}\cos x + \sin\frac{\pi}{4}\sin x$$

$$= 2\cos\frac{\pi}{4}\cos x = \frac{2}{\sqrt{2}}\cos x = \sqrt{2}\cos x$$

72. (d) : Area of triangle

$$= \frac{1}{2}|(-2)(5 + 1) + 1(-1 - 2) + 6(2 - 5)|$$

$$= \frac{1}{2}|(-2)(6) - 3 - 18| = \frac{1}{2}|-12 - 3 - 18|$$

$$= \frac{1}{2}|-33| = \frac{33}{2} \text{ sq. units}$$

73. (b) : Equation of line passing through (1, 0) and (–4, 1) is

$$\frac{y-0}{1-0}=\frac{x-1}{-4-1} \Rightarrow y=\frac{x-1}{-5}$$

or $x + 5y - 1 = 0$

Now, equation of line perpendicular to $x + 5y - 1 = 0$ is

$5x - y + \lambda = 0$ is for some constant λ

Also, $5x - y + \lambda = 0$ passes through (– 3, 5)

$\therefore$ $5(-3) - 5 + \lambda = 0 \Rightarrow \lambda = 20$

Hence, $5x - y + 20 = 0$ is the required equation of line

74. (b) : Given expansion is $(1 + x^2)^5 (1 + x)^4$

$= [{}^5C_0(x^2)^0 + {}^5C_1(x^2)^1 + {}^5C_2(x^2)^2 + {}^5C_3(x^2)^3 + {}^5C_4(x^2)^4 + {}^5C_5(x^2)^5]\ [{}^4C_0x^0 + {}^4C_1x + {}^4C_2x^2 + {}^4C_3x^3 + {}^4C_4x^4]$

$= [{}^5C_0 + {}^5C_1x^2 + {}^5C_2x^4 + {}^5C_3x^6 + {}^5C_4x^8 + {}^5C_5x^{10}]\ [{}^4C_0 + {}^4C_1x + {}^4C_2x^2 + {}^4C_3x^3 + {}^4C_4x^4]$

$\therefore$ Coefficient of $x^5 = {}^5C_1 \cdot {}^4C_3 + {}^5C_2 \cdot {}^4C_1$

$= 20 + 40 = 60$

75. (e) : Given expansion is

$(1 - 2x)^5 = {}^5C_0 - {}^5C_1(2x) + {}^5C_2(2x)^2 - {}^5C_3(2x)^3 + {}^5C_4(2x)^4 - {}^5C_5(2x)^5$

$\therefore$ Coefficient of $x^4 = {}^5C_4 . (2)^4 = 80$

76. (a) : We have, $5x^2 + y^2 + y = 8$

$$\Rightarrow 5x^2 + y^2 + y - 8 + \frac{1}{4} - \frac{1}{4} = 0$$

$$\Rightarrow (\sqrt{5}\,x)^2 + (y + 1/2)^2 - \frac{33}{4} = 0$$

$$\Rightarrow (\sqrt{5}\,x)^2 + (y + 1/2)^2 = \left(\frac{\sqrt{33}}{2}\right)^2$$

which represents an ellipse.

77. (e) : We have, $4x^2 + y^2 - 8x + 4y - 8 = 0$

$\Rightarrow 4(x^2 - 2x + 1 - 1) + (y^2 + 4y + 4 - 4) - 8 = 0$

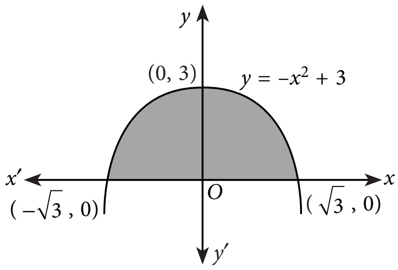
$\Rightarrow 4(x - 1)^2 - 4 + (y + 2)^2 - 4 - 8 = 0$

$$\Rightarrow \frac{(x-1)^2}{1/4} + (y + 2)^2 = 16$$

$$\Rightarrow \frac{(x-1)^2}{4} + \frac{(y+2)^2}{16} = 1$$

$\therefore$ Centre $\equiv$ (1, –2).

78. (c) :



$$\text{Required area} = 2\int_0^{\sqrt{3}} (-x^2+3)dx$$

$$= 2\left[\frac{-x^3}{3}+3x\right]_0^{\sqrt{3}} = 2\left[\frac{-3\sqrt{3}}{3}+3\sqrt{3}\right]$$

$= 2(2\sqrt{3}) = 4\sqrt{3}$

79. (a) : Order of differential equation

$$\left(\frac{d^3y}{dx^3}\right)^2 + \left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^5 = 0 \text{ is } 3.$$

80. (a) : We have, $f(x) = \sqrt{2x} + \dfrac{4}{\sqrt{2x}}$

$$\therefore f'(x) = \sqrt{2} \cdot \frac{1}{2\sqrt{x}} + \frac{4}{\sqrt{2}} \cdot \left(\frac{-1}{2x^{3/2}}\right)$$

$$= \frac{1}{\sqrt{2x}} - \frac{\sqrt{2}}{(x)^{3/2}}$$

$$\Rightarrow f'(2) = \frac{1}{2} - \frac{1}{2} = 0$$

81. (b) : Let r be the radius of given circle.

Given, area of circle $x^2 - 2x + y^2 - 10y + k = 0$ is 25π

i.e., $\pi r^2 = 25\pi \Rightarrow r^2 = 25$ ...(i)

Also, radius from the given equation is

$$r = \sqrt{(1)^2+(5)^2-k} \Rightarrow r^2 = 26 - k$$

$\Rightarrow 25 = 26 - k$ [Using (i)]

$\Rightarrow k = 1$

82. (d) : Let $I = \displaystyle\int_{2016}^{2017} \frac{\sqrt{x}}{\sqrt{x}+\sqrt{4033-x}}\,dx$ ...(i)

Also, $I = \displaystyle\int_{2016}^{2017} \frac{\sqrt{4033-x}}{\sqrt{4033-x}+\sqrt{x}}$ ...(ii)

Adding (i) and (ii), we get

$$2I = \int_{2016}^{2017} \frac{\left(\sqrt{x}+\sqrt{4033-x}\right)}{\left(\sqrt{x}+\sqrt{4033-x}\right)} dx = \int_{2016}^{2017} 1 \cdot dx$$

$$\Rightarrow \quad I = \frac{1}{2}[2017 - 2016] = \frac{1}{2}$$

83. (a) : We have, $\frac{dy}{dx} + y \tan x = \sec x$, $y(0) = 0$

This is a linear differential equation

$\therefore$ I.F. $= e^{\int \tan x \, dx} = e^{\log|\sec x|} = \sec x$

$\therefore$ Solution is given by $y \cdot \sec x = \int \sec x \cdot \sec x \, dx$

$\Rightarrow \; y \cdot \sec x = \int \sec^2 x \, dx \Rightarrow y \sec x = \tan x + c$

Now, we have $y(0) = 0$

$\Rightarrow \; (0) \cdot \sec 0 = \tan(0) + c \Rightarrow c = 0$

$\therefore$ Particular solution is, $y \sec x = \tan x$

84. (e) : Since the vectors $2\hat{i}+2\hat{j}+6\hat{k}$, $2\hat{i}+\lambda\hat{j}+6\hat{k}$, $2\hat{i}-3\hat{j}+\hat{k}$ are coplanar.

$$\therefore \begin{vmatrix} 2 & 2 & 6 \\ 2 & \lambda & 6 \\ 2 & -3 & 1 \end{vmatrix} = 0$$

$\Rightarrow \; 2(\lambda + 18) - 2(2 - 12) + 6(-6 - 2\lambda) = 0$

$\Rightarrow \; -10\lambda + 20 = 0 \Rightarrow \lambda = 2$

85. (b) : Distance of the point (2, 1, 0) from the plane $2x + y + 2z + 5 = 0$ is given by

$$\left|\frac{(2\cdot 2)+(1\cdot 1)+(0\cdot 2)+5}{\sqrt{2^2+1^2+2^2}}\right| = \left|\frac{4+1+5}{3}\right| = \frac{10}{3}$$

86. (e) : Coordinates of vertices of hyperbola $(0, \pm 15) = (0, \pm b)$

and foci $(0, \pm 20) = (0, \pm be)$

$\therefore \; b = 15$ and $be = 20$

$\Rightarrow \; e = \frac{20}{15} = \frac{4}{3}$

Now, $e^2 = 1 + \frac{a^2}{b^2} \Rightarrow \frac{16}{9} = 1 + \frac{a^2}{225}$

$\Rightarrow \; a^2 = \frac{7}{9} \times 225 = 175$

$\therefore$ Equation of hyperbola $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$

i.e., $\frac{y^2}{225} - \frac{x^2}{175} = 1$

87. (e) : Given, $\frac{15^3+6^3+3\cdot 6\cdot 15\cdot 21}{1+4(6)+6(36)+4(216)+1296}$

$$= \frac{(15+6)^3}{(1+6)^4} = \frac{(21)^3}{(7)^4} = \frac{27}{7}$$

88. (a) : The equation of the plane passing through the points (1, 0, 2), (–1, 1, 2) and (5, 0, 3) is

$$\begin{vmatrix} x-1 & y-0 & z-2 \\ -1-1 & 1-0 & 2-2 \\ 5-1 & 0-0 & 3-2 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} x-1 & y & z-2 \\ -2 & 1 & 0 \\ 4 & 0 & 1 \end{vmatrix} = 0$$

$\Rightarrow \; (x-1)(1) - y(-2) + (z-2)(-4)$

$\Rightarrow \; x - 1 + 2y - 4z + 8 = 0$

$\Rightarrow \; x + 2y - 4z + 7 = 0$

89. (b) : Given equation is $y^2 - 4y - x + 3 = 0$

$\Rightarrow \; (y-2)^2 - x - 1 = 0 \Rightarrow (y-2)^2 = x + 1$

Now, shifting the origin to the point (–1, 2) without rotating the axes and denoting the new coordinates w.r.t. these axes by X and Y, we get

$y - 2 = Y$ and $x + 1 = X$

Now, substituting $(X = 0, Y = 0)$

$y = 2, x = -1$

Hence, vertex w.r.t to old axes (–1, 2).

90. (a) : Given, $\vec{a}+\vec{b}+\vec{c}=\vec{0}$

$\Rightarrow \; \vec{a} = -(\vec{b}+\vec{c}) \Rightarrow |\vec{a}|^2 = |-(\vec{b}+\vec{c})|^2$

$\Rightarrow \; |\vec{a}|^2 = |\vec{b}|^2 + |\vec{c}|^2 + 2|\vec{b}||\vec{c}| \cos\theta$

(θ is angle between $\vec{b}$ and $\vec{c}$)

$\Rightarrow \; 49 = 25 + 9 + 30 \cos\theta$

$\Rightarrow \; \cos\theta = \frac{15}{30} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$

91. (d) : Let $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$

$\therefore \; f'(x) = 6x^2 - 18ax + 12a^2$

Now, $f'(x) = 0 \Rightarrow 6x^2 - 18ax + 12a^2 = 0$

$\Rightarrow \; (x - 2a)(x - a) = 0$

$\Rightarrow \; x = a$ or $2a$

Also, $f''(x) = 12x - 18a$

$\therefore \; f''(a) = 12a - 18a = -6a < 0 \qquad (\because a > 0)$

and $f''(2a) = 24a - 18a = 6a > 0$

Hence, $p = a$ and $q = 2a$

Now, $p^3 = q \Rightarrow a^3 = 2a \Rightarrow a(a^2 - 2) = 0$

$\Rightarrow \; a = \pm\sqrt{2}$

92. (e) : We have, $f(x) + f(y) = f(x + y)$

Put $x = y = 1$, we get

$f(1 + 1) = f(1) + f(1) \Rightarrow f(2) = 7 + 7 = 14$

$f(2 + 1) = f(2) + f(1) \Rightarrow f(3) = 14 + 7 = 21$

Continuing in the same way, we get
$f(4) = 28, f(5) = 35$ and so on.

$$\therefore \sum_{r=1}^{100} f(r) = f(1) + f(2) + f(3) + ... + f(100)$$

$$= 7 + 14 + 21 + ... + 700$$

Since, the series forms an A.P.

$$\therefore \sum_{r=1}^{100} f(r) = \frac{100}{2}[7 + 700] = 50 \times 707 = 50 \times 7 \times 101$$

93. (a) : Given, $\frac{(x-1)^2}{2} + \left(y + \frac{3}{4}\right)^2 = \frac{1}{16}$

or $\frac{(x-1)^2}{1/8} + \frac{(y+3/4)^2}{1/16} = 1$

$\therefore$ Eccentricity $(e) = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{(1/16)}{(1/8)}}$

$$= \sqrt{1 - \frac{1}{2}} = \frac{1}{\sqrt{2}}$$

94. (b) : We have, $\int_{-1}^{1} \max\{x, x^3\}\, dx$

$$= \int_{-1}^{0} x^3 dx + \int_{0}^{1} x\, dx = \left[\frac{x^4}{4}\right]_{-1}^{0} + \left[\frac{x^2}{2}\right]_{0}^{1} = \frac{1}{4}$$

95. (d) : We have, $\sin x + \cos y = 2$
It is possible only if $\sin x = 1$ and $\cos y = 1$

$\Rightarrow x = \frac{\pi}{2}$ and $y = 0$ $\therefore$ $x + y = \frac{\pi}{2} + 0 = \frac{\pi}{2}$

96. (a) : Given numbers $a, a + r, a + 2r$ are in A.P. Also their product = 64. This is possible only when three numbers are equal to 4.
i.e., $a = a + r = a + 2r = 4$
$\therefore$ Minimum value of $a + 2r$ is 4.

97. (b) : We have, $S = \frac{1}{9!} + \frac{1}{3!7!} + \frac{1}{5!5!} + \frac{1}{7!3!} + \frac{1}{9!}$

$$= \frac{1}{9!}\left[2 + \frac{2 \times 8 \times 9}{3!} + \frac{9 \times 8 \times 7 \times 6}{5!}\right]$$

$$= \frac{1}{9!}\left[\frac{2 \times 120 + 2(72)(20) + (3024)}{120}\right] = \frac{1 \times 6144}{10! \times 12} = \frac{2^9}{10!}$$

98. (d) : Given, $f(x) = \begin{vmatrix} x & x^2 & x^3 \\ 1 & 2x & 3x^2 \\ 0 & 2 & 6x \end{vmatrix}$

$= x(12x^2 - 6x^2) - x^2(6x) + x^3(2 - 0)$
$= 6x^3 - 6x^3 + 2x^3 = 2x^3$ $\therefore$ $f'(x) = 6x^2$

99. (c) : Let $I = \int \frac{x^2}{1 + (x^3)^2} dx$

Put $x^3 = t \Rightarrow 3x^2 dx = dt$

$\therefore$ $I = \frac{1}{3}\int \frac{dt}{1+t^2} = \frac{1}{3}[\tan^{-1} t] + c = \frac{1}{3}\tan^{-1}(x^3) + c$

100. (d) : We have, $f(x) = x^2 + e^x$
$\therefore$ $f'(x) = 2x + e^x$
$f''(x) = 2 + e^x$
$f'''(x) = e^x$
Hence, the least value of n so that $f_n = f_{n+1}$ is 3.

101. (e) : $\sin 765° = \sin(2 \times 360° + 45°)$

$$= \sin 45° = \frac{1}{\sqrt{2}}$$

102. (d) : Distance of the point $(3, -5)$ from the line $3x - 4y - 26 = 0$ is

$$\left|\frac{3 \cdot 3 + (-4)(-5) - 26}{\sqrt{3^2 + 4^2}}\right| = \left|\frac{9 + 20 - 26}{5}\right| = \frac{3}{5}$$

103. (c) : Given, $f(x) = \int_0^x (t^2 + t + 1)dt = \left[\frac{t^3}{3} + \frac{t^2}{2} + t\right]_0^x$

$$\therefore \quad f(x) = \left[\frac{x^3}{3} + \frac{x^2}{2} + x\right]$$

$\therefore$ $f(x) = \left[\frac{x^3}{3} + \frac{x^2}{2} + x\right]$ on $[2, 3]$ gives

$f(2) = \frac{8}{3} + \frac{4}{2} + 2 = \frac{20}{3}$ (minimum)

and $f(3) = \frac{27}{3} + \frac{9}{2} + 3 = \frac{33}{2}$ (maximum)

$\therefore$ Difference between the maximum and minimum value is $\frac{33}{2} - \frac{20}{3} = \frac{59}{6}$

104. (c) : Let, $f(x) = x^2 + ax + b = 0$
Since a and b are roots of $f(x)$
$\therefore$ $a + b = -a$ and $ab = b \Rightarrow a = 1$
$\therefore$ $1 + b = -1 \Rightarrow b = -2$
So, $f(x) = x^2 + x - 2$
Also, $f'(x) = 2x + 1$. For maximum/minimum $f'(x) = 0$

$\Rightarrow x = \frac{-1}{2}$

Now, $f''(x) = 2 > 0$

$\therefore\quad x = \frac{-1}{2}$ is the minimum point.

$\therefore\quad$ Minimum value $= f\left(-\frac{1}{2}\right) = \left(\frac{-1}{2}\right)^2 + \left(\frac{-1}{2}\right) - 2$

$= \frac{1}{4} - \frac{1}{2} - 2 = \frac{-9}{4}$

105. (b) : We have, $y = 4x + c$ and $\frac{x^2}{4} + y^2 = 1$

$\because\quad y = 4x + c$ touches the given ellipse

$\therefore\quad \frac{x^2}{4} + (4x + c)^2 = 1$

$\Rightarrow\quad x^2 + 4(16x^2 + c^2 + 8xc) = 4$

$\Rightarrow\quad x^2 + 64x^2 + 4c^2 + 32xc - 4 = 0$

$\Rightarrow\quad 65x^2 + 32xc + 4c^2 - 4 = 0$

Now, discriminant $D = 0 \Rightarrow (32c)^2 - 4(65)(4c^2 - 4) = 0$

$\Rightarrow\quad 1024c^2 - 1040c^2 + 1040 = 0$

$\Rightarrow\quad 16c^2 = 1040 \quad\Rightarrow\quad c^2 = 65$

$\Rightarrow\quad c = \pm\sqrt{65}$

106. (d) : Given equations $\lambda x - y = 2$, $2x - 3y = -\lambda$ and $3x - 2y = -1$ are consistent

$\therefore\quad \begin{vmatrix} \lambda & -1 & -2 \\ 2 & -3 & \lambda \\ 3 & -2 & 1 \end{vmatrix} = 0$

$\Rightarrow\quad \lambda(-3 + 2\lambda) + 1(2 - 3\lambda) - 2(-4 + 9) = 0$

$\Rightarrow\quad -3\lambda + 2\lambda^2 + 2 - 3\lambda + 8 - 18 = 0$

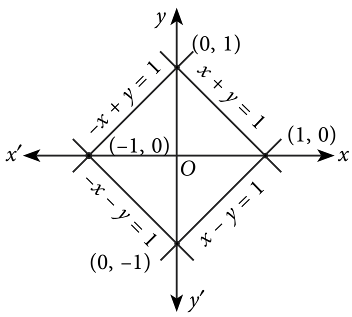
$\Rightarrow\quad \lambda^2 - 3\lambda - 4 = 0 \Rightarrow (\lambda - 4)(\lambda + 1) = 0$

$\therefore\quad \lambda = -1, 4$

107. (a) : The set $\{(x, y) : |x| + |y| = 1\}$ in xy plane represents

$$\begin{cases} x + y = 1; & x > 0, y > 0 \\ x - y = 1; & x > 0, y < 0 \\ -x + y = 1; & x < 0, y > 0 \\ -x - y = 1; & x < 0, y < 0 \end{cases}$$

i.e.,



Hence, the given set represents a square.

108. (a) : Let $\tan^{-1}\left(\frac{3}{4}\right) = \theta \Rightarrow \tan\theta = \frac{3}{4}$

$\therefore\quad \cos\theta = \frac{4}{5}$

Now, $\cos\left(\tan^{-1}\left(\frac{3}{4}\right)\right) = \cos\theta = \frac{4}{5}$

109. (b) : $\because\quad AB = BC$

$\therefore\quad$ By using distance formula, we have

$\sqrt{(1-6)^2 + (3+1)^2} = \sqrt{(x-1)^2 + (8-3)^2}$

On squaring both sides, we get

$(-5)^2 + (4)^2 = (x - 1)^2 + (5)^2$

$\Rightarrow\quad x^2 + 1 - 2x = 16 \Rightarrow x^2 - 2x - 15 = 0$

$\Rightarrow\quad (x + 3)(x - 5) = 0 \Rightarrow x = -3, 5$

110. (e) : We have, $n = 15$, Incorrect $\Sigma x^2 = 2830$, Incorrect $\Sigma(x) = 170$

$\therefore\quad$ Correct $\Sigma(x) =$ (Incorrect Σx – Incorrect value) + Correct value

$= (170 - 20) + 30 = 180$

$\therefore\quad$ Correct Mean = Correct $\frac{\Sigma x}{15} = \frac{180}{15} = 12$

Similarly, Correct Σx^2 = Incorrect Σx^2 – (Incorrect value)^2 + (Correct value)^2

$= 2830 - (20)^2 + (30)^2 = 2830 + 500 = 3330$

$\therefore\quad$ Correct variance = Correct $\frac{(\Sigma x^2)}{n}$ – (Correct mean)^2

$= \frac{3330}{15} - (12)^2 = 222 - 144 = 78$

111. (e) : Let θ be the angle between the lines

$\frac{x-2}{2} = \frac{y-1}{5} = \frac{z+3}{-3}$ and $\frac{x+2}{-1} = \frac{y-4}{8} = \frac{z-5}{4}$

$\therefore\quad \cos\theta = \frac{|2(-1) + 5(8) + (-3)(4)|}{\sqrt{(2)^2 + (5)^2 + (-3)^2}\sqrt{(-1)^2 + (8)^2 + (4)^2}}$

$= \frac{|-2 + 40 - 12|}{\sqrt{38}\sqrt{81}} = \frac{26}{9\sqrt{38}}$

i.e., $\theta = \cos^{-1}\left(\frac{26}{9\sqrt{38}}\right)$

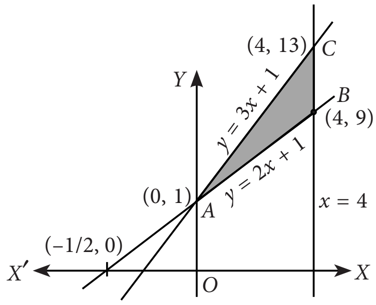
112. (d) : We have, $(\vec{x} - \vec{a})\cdot(\vec{x} + \vec{a}) = 12$

$\Rightarrow\quad |\vec{x}|^2 - |\vec{a}|^2 = 12$

Since, $\vec{a}$ be a unit vector $\therefore |\vec{a}| = 1$

$\Rightarrow\quad |\vec{x}|^2 - 1 = 12 \Rightarrow |\vec{x}|^2 = 13 \Rightarrow |\vec{x}| = \sqrt{13}$

113. (d) : Sides of triangular region are $y = 2x + 1$; $y = 3x + 1$ and $x = 4$.



$\therefore$ Area of shaded region *i.e.*, ΔABC where $A = (0, 1), B = (4, 9), C = (4, 13)$

$= \frac{1}{2}|0(9-13) + 4(13-1) + 4(1-9)| = \frac{1}{2}|4(12) - 32|$

$= \frac{1}{2} \times 16 = 8$ sq. units

114. (b) : Given, ${}^nC_{r-1} = 36, {}^nC_r = 84, {}^nC_{r+1} = 126$

Since, $\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}$

$\therefore \frac{84}{36} = \frac{n-r+1}{r} \Rightarrow \frac{n-r+1}{r} = \frac{7}{3}$

$\Rightarrow 7r = 3n - 3r + 3 \Rightarrow 10r = 3n + 3$ ...(i)

Also, $\frac{{}^nC_r}{{}^nC_{r+1}} = \frac{r+1}{n-r} \Rightarrow \frac{84}{126} = \frac{r+1}{n-r} \Rightarrow \frac{r+1}{n-r} = \frac{2}{3}$

$\Rightarrow 3r + 3 = 2n - 2r \Rightarrow 5r = 2n - 3$ ...(ii)

Solving (i) & (ii), we get $r = 3$

115. (e) : $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h\to 0} \frac{f(x)f(h) - f(x)}{h}$ $[\because f(x+y) = f(x)f(y)]$

$= f(x) \lim_{h\to 0} \frac{f(h)-1}{h} = f(x) \lim_{h\to 0} \frac{1+\sin 3h\, g(h) - 1}{h}$

$= f(x) \lim_{h\to 0} g(h) \frac{\sin 3h}{h}$

$= f(x)g(0) \lim_{h\to 0} \frac{\sin 3h}{3h} \times 3 = 3 f(x)\, g(0)$

116. (b) : Given, $\begin{vmatrix} x-1 & 1 & 1 \\ 1 & x-1 & 1 \\ 1 & 1 & x-1 \end{vmatrix} = 0$

$\Rightarrow (x-1)[(x-1)^2 - 1] - 1[x - 1 - 1] + 1[1 - x + 1] = 0$

$\Rightarrow (x-1)[x^2 + 1 - 2x - 1] - x + 2 + 2 - x = 0$

$\Rightarrow (x-1)\, x(x-2) - 2x + 4 = 0$

$\Rightarrow x(x-1)(x-2) - 2(x-2) = 0$

$\Rightarrow (x-2)[x^2 - x - 2] = 0 \Rightarrow (x-2)(x-2)(x+1) = 0$

$\Rightarrow x = 2, -1$

117. (None of the options is correct) :

Given, $T_7 = T_8$

$\Rightarrow {}^nC_6(2a)^{n-6}(-3b)^6 = {}^nC_7(2a)^{n-7}(-3b)^7$

$\Rightarrow \frac{n!}{(n-6)!6!}(2a)^{n-6}(-3b)^6 = \frac{n!}{(n-7)!7!}(2a)^{n-7}(-3b)^7$

$\Rightarrow \frac{1}{(n-6)}(2a) = \frac{(-3b)}{7} \Rightarrow \frac{2a}{3b} = \frac{-(n-6)}{7}$

Applying componendo & dividendo, we get

$\frac{2a+3b}{2a-3b} = \frac{6-n+7}{6-n-7} = \frac{13-n}{-n-1} = \frac{n-13}{n+1}$

118. (c) : Mean of first n odd natural numbers

$\bar{x} = \frac{1+3+5+....+(2n-1)}{n} = \frac{n^2}{n} = n$

Sum of square of first n odd natural numbers *i.e.*,

$1^2 + 3^2 + 5^2 + ... + (2n-1)^2 = \frac{n(2n+1)(2n-1)}{3}$

$\therefore$ Standard deviation $= \sqrt{\frac{n(2n+1)(2n-1)}{3n} - n^2}$

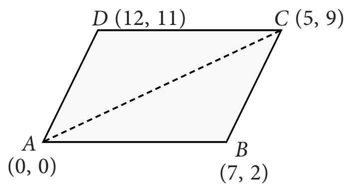
$= \sqrt{\frac{n^2-1}{3}}$

119. (b) : Given, $S = \{1, 2, 3, ..., 10\}$.

$\therefore$ Set containing odd numbers of $S = \{1, 3, 5, 7, 9\}$

$\therefore$ Number of subsets of S containing only odd numbers $= (2)^5 - 1 = 32 - 1 = 31$

120. (e) : Let the vertices of parallelogram be $A(0, 0)$, $B(7, 2)$, $C(5, 9)$ and $D(12, 11)$



Area of || gm $ABCD$ = area of ΔABC + area of ΔADC

Now, area of $\Delta ABC = \frac{1}{2}|0(2-9) + 7(9-0) + 5(0-2)|$

$= \frac{1}{2}|63 - 10| = \frac{53}{2}$

$\therefore$ Area of $\Delta ADC = \frac{1}{2}|0(11-9) + 12(9-0) + 5(0-11)|$

$= \frac{1}{2}|108 - 55| = \frac{53}{2}$

$\therefore$ Area of || gm $ABCD = \frac{53}{2} + \frac{53}{2} = 53$ sq. units

SOLVED PAPER **2017**

Karnataka CET

1. The distance of the point (–2, 4, –5) from the line $\frac{x+3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$ is

(a) $\frac{\sqrt{37}}{10}$ (b) $\frac{37}{\sqrt{10}}$ (c) $\sqrt{\frac{37}{10}}$ (d) $\frac{37}{10}$

2. If A is a square matrix of order 3 × 3, then $|KA|$ is equal to

(a) $K|A|$ (b) $K^2|A|$
(c) $3K|A|$ (d) $K^3|A|$

3. Equation of line passing through the point (1, 2) and perpendicular to the line $y = 3x - 1$ is

(a) $x - 3y = 0$ (b) $x + 3y = 0$
(c) $x + 3y - 7 = 0$ (d) $x + 3y + 7 = 0$

4. General solution of differential equation $\frac{dy}{dx}+y=1(y\neq 1)$ is

(a) $\log\left|\frac{1}{1-y}\right|=x+C$ (b) $\log|1-y|=x+C$

(c) $\log|1+y|=x+C$ (d) $\log\left|\frac{1}{1-y}\right|=-x+C$

5. The value of C in mean value theorem for the function $f(x) = x^2$ in [2, 4] is

(a) 2 (b) 4 (c) $\frac{7}{2}$ (d) 3

6. The value of $\lim_{\theta\to 0}\frac{1-\cos 4\theta}{1-\cos 6\theta}$ is

(a) $\frac{9}{4}$ (b) $\frac{4}{9}$ (c) $\frac{9}{3}$ (d) $\frac{3}{4}$

7. If $y=\tan^{-1}\left(\frac{\sin x+\cos x}{\cos x-\sin x}\right)$, then $\frac{dy}{dx}$ is equal to

(a) 0 (b) $\frac{1}{2}$ (c) $\frac{\pi}{4}$ (d) 1

8. If $\left(\frac{1+i}{1-i}\right)^m=1$, then the least positive integral value of m is

(a) 4 (b) 1 (c) 2 (d) 3

9. $\int_{-5}^{5}|x+2|\,dx$ is equal to

(a) 28 (b) 29 (c) 27 (d) 30

10. $\int\frac{\cos 2x-\cos 2\theta}{\cos x-\cos\theta}dx$ is equal to

(a) $2(\sin x + x\cos\theta) + C$
(b) $2(\sin x - x\cos\theta) + C$
(c) $2(\sin x + 2x\cos\theta) + C$
(d) $2(\sin x - 2x\cos\theta) + C$

11. The area of the region bounded by the curve $y = x^2$ and the line $y = 16$ is

(a) $\frac{256}{3}$ sq. units (b) $\frac{128}{3}$ sq. units

(c) $\frac{32}{3}$ sq. units (d) $\frac{64}{3}$ sq. units

12. If A and B are finite sets and $A \subset B$, then

(a) $n(A \cup B) = n(B)$ (b) $n(A \cap B) = n(B)$
(c) $n(A \cap B) = \phi$ (d) $n(A \cup B) = n(A)$

13. If a matrix A is both symmetric and skew symmetric, then

(a) A is diagonal matrix
(b) A is a zero matrix
(c) A is scalar matrix
(d) A is square matrix

14. If $\begin{vmatrix}3 & x\\ x & 1\end{vmatrix}=\begin{vmatrix}3 & 2\\ 4 & 1\end{vmatrix}$ then x is equal to

(a) 8 (b) 4
(c) $\pm 2\sqrt{2}$ (d) 2

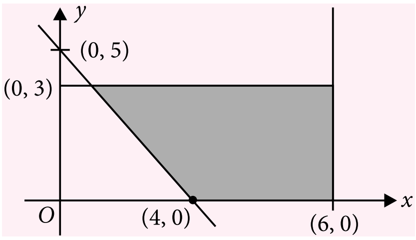
15. The integrating factor of the differential equation $x \cdot \frac{dy}{dx} + 2y = x^2$ is $(x \neq 0)$

(a) $e^{\log x}$ (b) $\log|x|$
(c) x (d) x^2

16. The perpendicular distance of the point $P(6, 7, 8)$ from XY-plane is

(a) 7 (b) 6 (c) 8 (d) 5

17. The shaded region in the figure is the solution set of the inequations



(a) $5x + 4y \leq 20, x \leq 6, y \leq 3, x \geq 0, y \geq 0$
(b) $5x + 4y \geq 20, x \leq 6, y \geq 3, x \geq 0, y \geq 0$
(c) $5x + 4y \geq 20, x \leq 6, y \leq 3, x \geq 0, y \geq 0$
(d) $5x + 4y \geq 20, x \geq 6, y \leq 3, x \geq 0, y \geq 0$

18. If an LPP admits optimal solution at two consecutive vertices of a feasible region, then

(a) the required optimal solution is at the midpoint of the line joining two points.
(b) the optimal solution occurs at every point on the line joining these two points.
(c) the LPP under consideration is not solvable.
(d) the LPP under consideration must be reconstructed.

19. $3 + 5 + 7 + \ldots$ to n terms is

(a) n^2 (b) $n(n - 2)$
(c) $n(n + 2)$ (d) $(n + 1)^2$

20. If $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$, then the value of x and y are

(a) $x = 3, y = 3$ (b) $x = -3, y = 3$
(c) $x = 3, y = -3$ (d) $x = -3, y = -3$

21. The derivative of $\cos^{-1}(2x^2 - 1)$ w.r.t $\cos^{-1} x$ is

(a) 2 (b) $\frac{2}{x}$
(c) $1 - x^2$ (d) $\frac{-1}{2\sqrt{1-x^2}}$

22. A box has 100 pens of which 10 are defective. The probability that out of a sample of 5 pens drawn one by one with replacement and atmost one is defective is

(a) $\frac{9}{10}$ (b) $\frac{1}{2}\left(\frac{9}{10}\right)^4$
(c) $\left(\frac{9}{10}\right)^5 + \frac{1}{2}\left(\frac{9}{10}\right)^4$ (d) $\frac{1}{2}\left(\frac{9}{10}\right)^5$

23. If $y = \log(\log x)$, then $\frac{d^2y}{dx^2}$ is equal to

(a) $\frac{(1+\log x)}{x^2 \log x}$ (b) $\frac{-(1+\log x)}{(x \log x)^2}$
(c) $\frac{(1+\log x)}{(x \log x)^2}$ (d) $\frac{-(1+\log x)}{x^2 \log x}$

24. $\int \frac{(x+3)e^x}{(x+4)^2} dx$ is equal to

(a) $\frac{e^x}{(x+4)} + C$ (b) $\frac{e^x}{(x+4)^2} + C$
(c) $\frac{e^x}{(x+3)} + C$ (d) $\frac{1}{(x+4)^2} + C$

25. $\int_0^{\pi/2} \frac{1}{a^2 \sin^2 x + b^2 \cos^2 x} dx$ is equal to

(a) $\frac{\pi a}{4b}$ (b) $\frac{\pi b}{4a}$ (c) $\frac{\pi}{2ab}$ (d) $\frac{\pi a}{2b}$

26. Let $f: R \rightarrow R$ be defined by $f(x) = x^4$, then

(a) f is one-one but not onto
(b) f is neither one-one nor onto
(c) f is one-one and onto
(d) f may be one-one and onto

27. The point on the curve $y^2 = x$ where the tangent makes an angle $\frac{\pi}{4}$ with X-axis is

(a) $(1, 1)$ (b) $\left(\frac{1}{4}, \frac{1}{2}\right)$
(c) $\left(\frac{1}{2}, \frac{1}{4}\right)$ (d) $(4, 2)$

28. The total number of terms in the expansion of $(x + a)^{47} - (x - a)^{47}$ after simplification is

(a) 24 (b) 96 (c) 47 (d) 48

29. The function $f(x) = x^2 + 2x - 5$ is strictly increasing in the interval
(a) $[-1, \infty)$ (b) $(-\infty, -1)$
(c) $(-\infty, -1]$ (d) $(-1, \infty)$

30. The degree of the differential equation
$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^2 = \frac{d^2y}{dx^2} \text{ is}$$
(a) 1 (b) 4 (c) 2 (d) 3

31. Binary operation $*$ on $R - \{-1\}$ defined by
$$a * b = \frac{a}{b+1} \text{ is}$$
(a) $*$ is associative and commutative
(b) $*$ is neither associative nor commutative
(c) $*$ is commutative but not associative
(d) $*$ is associative but not commutative

32. The plane $2x - 3y + 6z - 11 = 0$ makes an angle $\sin^{-1}(\alpha)$ with X-axis. The value of α is equal to
(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{2}{7}$ (c) $\frac{\sqrt{2}}{3}$ (d) $\frac{3}{7}$

33. If coefficient of variation is 60 and standard deviation is 24, then arithmetic mean is
(a) $\frac{20}{7}$ (b) $\frac{7}{20}$ (c) $\frac{1}{40}$ (d) 40

34. The contrapositive statement of the statement "If x is prime number, then x is odd" is
(a) If x is not a prime number, then x is odd.
(b) If x is not a prime number, then x is not odd.
(c) If x is a prime number, then x is not odd.
(d) If x is not odd, then x is not a prime number.

35. The probability distribution of X is

X	0	1	2	3
$P(X)$	0.3	k	$2k$	$2k$

The value of k is
(a) 0.7 (b) 0.3 (c) 1 (d) 0.14

36. $\int \sqrt{x^2+2x+5}\,dx$ is equal to
(a) $(x+1)\sqrt{x^2+2x+5}$
$$-2\log\left|x+1+\sqrt{x^2+2x+5}\right|+C$$
(b) $\frac{1}{2}(x+1)\sqrt{x^2+2x+5}$
$$+2\log\left|x+1+\sqrt{x^2+2x+5}\right|+C$$
(c) $(x+1)\sqrt{x^2+2x+5}$
$$+2\log\left|x+1+\sqrt{x^2+2x+5}\right|+C$$
(d) $(x+1)\sqrt{x^2+2x+5}$
$$+\frac{1}{2}\log\left|x+1+\sqrt{x^2+2x+5}\right|+C$$

37. If ${}^nC_{12} = {}^nC_8$ then n is equal to
(a) 12 (b) 26 (c) 6 (d) 20

38. If $y = \begin{vmatrix} f(x) & g(x) & h(x) \\ l & m & n \\ a & b & c \end{vmatrix}$, then $\frac{dy}{dx}$ is equal to

(a) $\begin{vmatrix} f'(x) & g'(x) & h'(x) \\ l & m & n \\ a & b & c \end{vmatrix}$

(b) $\begin{vmatrix} l & m & n \\ f'(x) & g'(x) & h'(x) \\ a & b & c \end{vmatrix}$

(c) $\begin{vmatrix} f'(x) & l & a \\ g'(x) & m & b \\ h'(x) & n & c \end{vmatrix}$ (d) $\begin{vmatrix} l & m & n \\ a & b & c \\ f'(x) & g'(x) & h'(x) \end{vmatrix}$

39. If $\tan^{-1}x + \tan^{-1}y = \frac{4\pi}{5}$, then $\cot^{-1}x + \cot^{-1}y$ is equal to
(a) $\frac{\pi}{5}$ (b) $\frac{3\pi}{5}$ (c) $\frac{2\pi}{5}$ (d) π

40. The range of the function $f(x) = \sqrt{9-x^2}$ is
(a) $[0, 3]$ (b) $(0, 3]$
(c) $(0, 3)$ (d) $[0, 3)$

41. Two events A and B will be independent if
(a) $P(A' \cap B') = (1 - P(A))\,(1 - P(B))$
(b) A and B are mutually exclusive
(c) $P(A) + P(B) = 1$
(d) $P(A) = P(B)$

42. The eccentricity of the ellipse $\frac{x^2}{36} + \frac{y^2}{16} = 1$ is
(a) $\frac{2\sqrt{5}}{6}$ (b) $\frac{2\sqrt{13}}{4}$ (c) $\frac{2\sqrt{5}}{4}$ (d) $\frac{2\sqrt{13}}{6}$

43. If $\vec{a}$ & $\vec{b}$ are unit vectors, then angle between $\vec{a}$ and $\vec{b}$ for $\sqrt{3}\vec{a} - \vec{b}$ to be unit vector is
(a) 45° (b) 30° (c) 90° (d) 60°

44. If $\vec{a} = 2\hat{i} + \lambda\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$ are orthogonal, then value of λ is

(a) $\frac{3}{2}$ (b) 1 (c) $-\frac{5}{2}$ (d) 0

45. The value of $\cos^2 45° - \sin^2 15°$ is

(a) $\frac{\sqrt{3}+1}{2\sqrt{2}}$ (b) $\frac{\sqrt{3}}{4}$

(c) $\frac{\sqrt{3}-1}{2\sqrt{2}}$ (d) $\frac{\sqrt{3}}{2}$

46. The range of $\sec^{-1}x$ is

(a) $[0, \pi]$ (b) $[0, \pi] - \left\{\frac{\pi}{2}\right\}$

(c) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ (d) $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

47. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then the value of $\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}$ is equal to

(a) $-\frac{3}{2}$ (b) 3 (c) $\frac{3}{2}$ (d) 1

48. $\int_{-\pi/2}^{\pi/2} \frac{dx}{e^{\sin x}+1}$ is equal to

(a) 1 (b) 0 (c) $\frac{\pi}{2}$ (d) $-\frac{\pi}{2}$

49. The rate of change of volume of a sphere with respect to its surface area when the radius is 4 cm is

(a) 2 cm^3/cm^2 (b) 4 cm^3/cm^2
(c) 8 cm^3/cm^2 (d) 6 cm^3/cm^2

50. $\int_0^{\pi/2} \frac{\tan^7 x}{\cot^7 x + \tan^7 x}\,dx$ is equal to

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{3}$

51. If $|x-2| \le 1$, then

(a) $x \in (1, 3)$ (b) $x \in (-1, 3)$
(c) $x \in [1, 3]$ (d) $x \in [-1, 3)$

52. $\int_{0.2}^{3.5} [x]dx$ is equal to

(a) 3.5 (b) 4.5 (c) 3 (d) 4

53. The area of triangle with vertices $(K, 0)$, $(4, 0)$, $(0, 2)$ is 4 square units, then value of K is

(a) 8 (b) 0 or –8 (c) 0 (d) 0 or 8

54. If $f(x) = \begin{cases} Kx^2 & \text{if } x \le 2 \\ 3 & \text{if } x > 2 \end{cases}$ is continuous at $x = 2$, then the value of K is

(a) $\frac{4}{3}$ (b) $\frac{3}{4}$ (c) 3 (d) 4

55. If $A = \frac{1}{\pi}\begin{bmatrix} \sin^{-1}(\pi x) & \tan^{-1}\left(\frac{x}{\pi}\right) \\ \sin^{-1}\left(\frac{x}{\pi}\right) & \cot^{-1}(\pi x) \end{bmatrix}$

$B = \begin{bmatrix} -\cos^{-1}(\pi x) & \tan^{-1}\left(\frac{x}{\pi}\right) \\ \sin^{-1}\left(\frac{x}{\pi}\right) & -\tan^{-1}(\pi x) \end{bmatrix}$

then $A - B$ is equal to

(a) $\frac{1}{2}I$ (b) I (c) O (d) $2I$

56. If $f(x) = 8x^3$, $g(x) = x^{1/3}$, then $fog(x)$ is

(a) 8^3x (b) $8x^3$
(c) $8x$ (d) $(8x)^{1/3}$

57. Let $\Delta = \begin{vmatrix} Ax & x^2 & 1 \\ By & y^2 & 1 \\ Cz & z^2 & 1 \end{vmatrix}$ and $\Delta_1 = \begin{vmatrix} A & B & C \\ x & y & z \\ zy & zx & xy \end{vmatrix}$ then

(a) $\Delta_1 = -\Delta$ (b) $\Delta_1 = \Delta$
(c) $\Delta_1 = 2\Delta$ (d) $\Delta_1 \ne \Delta$

58. If $\sin x = \frac{2t}{1+t^2}$, $\tan y = \frac{2t}{1-t^2}$, then $\frac{dy}{dx}$ is equal to

(a) 1 (b) –1 (c) 2 (d) 0

59. Reflection of the point (α, β, γ) in XY plane is

(a) $(0, 0, \gamma)$ (b) $(\alpha, \beta, -\gamma)$
(c) $(-\alpha, -\beta, \gamma)$ (d) $(\alpha, \beta, 0)$

60. Area of the region bounded by the curve $y = \cos x$, $x = 0$ and $x = \pi$ is

(a) 2 sq. units (b) 3 sq. units
(c) 4 sq. units (d) 1 sq. units

MPP-2 CLASS XI ANSWER KEY

1.	(c)	2.	(a)	3.	(d)	4.	(a)	5.	(b)
6.	(c)	7.	(b)	8.	(a,c)	9.	(b,c)	10.	(d)
11.	(b,c)	12.	(a,c,d)	13.	(a)	14.	(a)	15.	(b)
16.	(d)	17.	(4)	18.	(5)	19.	(1)	20.	(1)

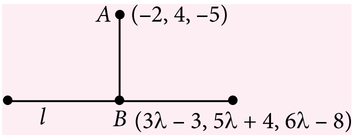
SOLUTIONS

1. (c) : Given, point is $A(-2, 4, -5)$,

Line (l) is $\dfrac{x+3}{3}=\dfrac{y-4}{5}=\dfrac{z+8}{6}=\lambda$ (say)

Co-ordinates of B are $(3\lambda - 3, 5\lambda + 4, 6\lambda - 8)$

$\therefore$ Direction ratios of AB are $(3\lambda - 1, 5\lambda, 6\lambda - 3)$



Now, $(3\lambda - 1)(3) + (5\lambda)(5) + (6\lambda - 3)(6) = 0$

$\Rightarrow \lambda = \dfrac{3}{10}$

$\therefore \overrightarrow{AB} = -\dfrac{1}{10}\hat{i}+\dfrac{3}{2}\hat{j}-\dfrac{6}{5}\hat{k}$

$\therefore d = |\overrightarrow{AB}| = \sqrt{\dfrac{1}{100}+\dfrac{9}{4}+\dfrac{36}{25}}$

$= \sqrt{\dfrac{1+225+144}{100}} = \sqrt{\dfrac{370}{100}} = \sqrt{\dfrac{37}{10}}$

2. (d): We have $|KA| = K^n|A|$, Here $n = 3$

$\therefore |KA| = K^3|A|$

3. (c) : Equation of required line is $y - 2 = -\dfrac{1}{3}(x - 1)$

$\Rightarrow x + 3y - 7 = 0$

4. (a) : Given, $\dfrac{dy}{dx}+y=1$

This is a linear differential equation.

$\therefore$ I.F. $= e^{\int 1dx} = e^x$

$\therefore$ Solution is given by, $ye^x = \int e^x \cdot 1\ dx = e^x + C_1$

$\Rightarrow e^x(y - 1) = C_1 \Rightarrow x + \log|y - 1| = \log C_1$

$\Rightarrow -x - \log|y - 1| = -\log C_1$

$\Rightarrow \log\left|\dfrac{1}{y-1}\right| = x + C$ [where $-\log C_1 = C$]

or $\log\left|\dfrac{1}{1-y}\right| = x + C$

5. (d): We have, $f(x) = x^2$ in $[2,4]$

$\therefore$ According to mean value theorem,

We have, $f'(C) = \dfrac{f(b)-f(a)}{b-a}$

[where $a = 2$ and $b = 4$]

$\therefore 2C = \dfrac{f(4)-f(2)}{4-2} = \dfrac{(4)^2-(2)^2}{2} = \dfrac{12}{2}$

$\Rightarrow 2C = 6 \Rightarrow C = 3$

6. (b): We have, $\lim\limits_{\theta\to 0}\dfrac{1-\cos 4\theta}{1-\cos 6\theta}$

$= \lim\limits_{\theta\to 0}\dfrac{\sin^2 2\theta}{\sin^2 3\theta} = \dfrac{(2)^2}{(3)^2} = \dfrac{4}{9}$

7. (d): We have, $y = \tan^{-1}\left(\dfrac{\sin x+\cos x}{\cos x-\sin x}\right)$

$= \tan^{-1}\left(\dfrac{1+\tan x}{1-\tan x}\right) = \tan^{-1}\left(\tan\left(\dfrac{\pi}{4}+x\right)\right) = \dfrac{\pi}{4}+x$

$\therefore \dfrac{dy}{dx} = 1$

8. (a) : Given, $\left(\dfrac{1+i}{1-i}\right)^m = 1 \Rightarrow i^m = i^4 \Rightarrow m = 4$

9. (b): Let $I = \int\limits_{-5}^{5}|x+2|\,dx$

$= -\int\limits_{-5}^{-2}(x+2)dx + \int\limits_{-2}^{5}(x+2)dx$

$= -\left[\dfrac{x^2}{2}+2x\right]_{-5}^{-2} + \left[\dfrac{x^2}{2}+2x\right]_{-2}^{5}$

$= \dfrac{9}{2}+\dfrac{49}{2} = \dfrac{58}{2} = 29$

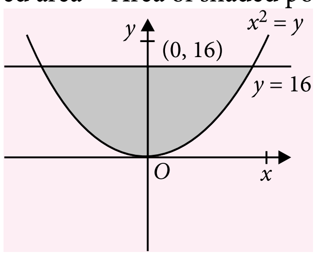
10. (a) : Let $I = \int \dfrac{\cos 2x-\cos 2\theta}{\cos x-\cos\theta}dx$

$= \int \dfrac{(2\cos^2 x-1)-(2\cos^2\theta-1)}{\cos x-\cos\theta}\,dx$

$= \int \dfrac{2(\cos^2 x-\cos^2\theta)}{\cos x-\cos\theta}dx = 2\int(\cos x+\cos\theta)dx$

$= 2(\sin x + x\cos\theta) + C$

11. (a) : Required area = Area of shaded portion



$$= 2\int_0^{16} \sqrt{y}\,dy = 2\cdot\frac{2}{3}\left[y^{3/2}\right]_0^{16} = \frac{4}{3}[4^3] = \frac{256}{3} \text{ sq. units}$$

12. (a) : Given, $A \subset B \Rightarrow A \cap B = A$

$\therefore\quad n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= n(A) + n(B) - n(A) = n(B)$

13. (b): Given, A is symmetric $\Rightarrow a_{ij} = a_{ji}$ ...(i) $i \neq j$

A is skew symmetric $\Rightarrow a_{ij} = -a_{ji}$ ...(ii) and $a_{ii} = 0$

Adding (i) and (ii) we get $2a_{ij} = 0 \Rightarrow a_{ij} = 0$

$\therefore$ A is a zero matrix.

14. (c) : We have, $\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$

$\Rightarrow\quad 3 - x^2 = 3 - 8 \Rightarrow\quad x^2 = 8 \Rightarrow x = \pm 2\sqrt{2}$

15. (d): Given, $x\cdot\frac{dy}{dx} + 2y = x^2 \Rightarrow \frac{dy}{dx} + \frac{2y}{x} = x$

$\therefore\quad \text{I.F.} = e^{\int \frac{2}{x}dx} = e^{\log x^2} = x^2$

16. (c) : Perpendicular distance of the point $P(6,7,8)$ from XY-plane is 8.

17. (c) : Clearly, shaded region represents

$5x + 4y \geq 20, x \leq 6, y \leq 3, x \geq 0, y \geq 0$

18. (b): The optimal solution occurs at every point on the line joining these two points.

19. (c) : Given series is in A.P. with first term $(a) = 3$, common difference $(d) = 2$

$\therefore\quad S_n = \frac{n}{2}[2\times 3 + (n-1)2] = n(n+2)$

20. (a) : Given $\begin{bmatrix} 2+y & 6 \\ 1 & 2x+2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

$\Rightarrow\quad 2 + y = 5$ and $2x + 2 = 8 \Rightarrow y = 3, x = 3$

21. (a) : Let $u = \cos^{-1}(2x^2 - 1)$ and $v = \cos^{-1} x \Rightarrow \cos v = x$

$\therefore\quad u = \cos^{-1}(2\cos^2 v - 1) = \cos^{-1}(\cos 2v) = 2v$

$\therefore\quad \frac{du}{dv} = 2$

22. (c) : $p = \frac{10}{100} = \frac{1}{10}, q = 1 - p = \frac{9}{10}, n = 5$

$$P(X = x) = {}^5C_x\left(\frac{1}{10}\right)^x\left(\frac{9}{10}\right)^{5-x}$$

$\therefore\quad P(X \leq 1) = P(X = 0) + P(X = 1)$

$$= {}^5C_0\left(\frac{1}{10}\right)^0\left(\frac{9}{10}\right)^5 + {}^5C_1\left(\frac{1}{10}\right)^1\left(\frac{9}{10}\right)^4$$

23. (b): We have, $y = \log(\log x) \Rightarrow \frac{dy}{dx} = \frac{1}{x\log x}$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-1}{(x\log x)^2}\left(x\cdot\frac{1}{x} + \log x\cdot 1\right) = \frac{-(1+\log x)}{(x\log x)^2}$$

24. (a) : Let $I = \int \frac{(x+3)e^x}{(x+4)^2}dx = \int \frac{(x+4-1)e^x}{(x+4)^2}dx$

$$= \int\left[\frac{1}{x+4} - \frac{1}{(x+4)^2}\right]e^x dx = \frac{e^x}{x+4} + C$$

$$\left[\because \int e^x(f(x) + f'(x))dx = e^x f(x) + C\right]$$

25. (c) : Let $I = \int_0^{\pi/2} \frac{1}{a^2\sin^2 x + b^2\cos^2 x}dx$

$$= \int_0^{\pi/2} \frac{\sec^2 x\,dx}{a^2\tan^2 x + b^2}$$

Put $\tan x = t \Rightarrow \sec^2 x\,dx = dt$

$$\therefore\quad I = \int_0^{\infty} \frac{dt}{a^2t^2 + b^2} = \int_0^{\infty} \frac{dt}{a^2\left(t^2 + \frac{b^2}{a^2}\right)}$$

$$= \frac{1}{a^2}\left[\frac{a}{b}\tan^{-1}\frac{at}{b}\right]_0^{\infty} = \frac{\pi}{2ab}$$

26. (b): Given $f(x) = x^4$

Now, $f(1) = f(-1)$ but $1 \neq -1$

$\therefore$ f is not one-one

Also, co-domain of f is R and range of f is $[0, \infty)$

$\therefore$ f is not onto

27. (b): We have, $y^2 = x \Rightarrow 2yy' = 1$

$\Rightarrow y' = \frac{1}{2y} = \tan\frac{\pi}{4} \Rightarrow y = \frac{1}{2}$

When $y = \frac{1}{2}, x = y^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

28. (a) : Number of terms in $(x + a)^{47} - (x - a)^{47}$

$= \frac{47+1}{2} = 24$

[Number of terms in $(a + b)^n - (a - b)^n$ is $\frac{n+1}{2}$, when n is odd number]

29. (d): f is strictly increasing $\Rightarrow f'(x) > 0$

$\Rightarrow \quad 2x + 2 > 0 \;\Rightarrow x > -1$

30. (a): Highest order derivative is $\frac{d^2y}{dx^2}$ and its power is 1.

Hence, degree of differential equation is 1.

31. (b): We have, $a * b = \frac{a}{b+1}$

$1 * 2 = \frac{1}{3}$ but $2 * 1 = \frac{2}{2} = 1$

Thus $1 * 2 \neq 2 * 1 \quad \therefore\ *$ is not commutative

Now, $(1 * 2) * 3 = \frac{1}{3} * 3 = \frac{1/3}{4} = \frac{1}{12}$

and $1 * (2 * 3) = 1 * \frac{1}{2} = \frac{1}{\frac{1}{2}+1} = \frac{2}{3}$

$\therefore\ *$ is not associative

[Infact, $*$ is not a binary operation !!!]

32. (b): Let ϕ be the angle made by plane $2x - 3y + 6z - 11 = 0$ and X-axis *i.e.*, (1,0,0)

$\therefore \quad \sin\phi = \frac{|2\times 1 - 3\times 0 + 6\times 0|}{\sqrt{4+9+36}\sqrt{1+0+0}}$

$\Rightarrow \quad \phi = \sin^{-1}\left(\frac{2}{7}\right) = \sin^{-1}(\alpha)$

33. (d): We have C.V. $= \frac{\sigma}{\bar{x}} \times 100$

$\Rightarrow \quad 60 = \frac{24}{\bar{x}} \times 100 \Rightarrow \bar{x} = 40$

34. (d): The contrapositive statement of the statement "If x is prime number, then x is odd" is "If x is not odd, then x is not a prime number."

35. (d): We know, $\sum P(X) = 1$

$\Rightarrow \quad 0.3 + k + 2k + 2k = 1 \;\Rightarrow 5k = 0.7 \;\Rightarrow 0.14$

36. (b): Let $I = \int\sqrt{x^2+2x+5}\,dx = \int\sqrt{(x+1)^2+2^2}\,dx$

$= \frac{(x+1)}{2}\sqrt{x^2+2x+5} + 2\log\left|x+1+\sqrt{x^2+2x+5}\right| + C$

37. (d): Given, ${}^nC_{12} = {}^nC_8 \;\Rightarrow 8 + 12 = n \;\Rightarrow n = 20$

$[\because\ {}^nC_x = {}^nC_y \Rightarrow x = y \text{ or } x + y = n]$

38. (a, c, d)

39. (a): Given, $\tan^{-1}x + \tan^{-1}y = \frac{4\pi}{5}$

$\Rightarrow \quad \cot^{-1}x + \cot^{-1}y = \pi - \frac{4\pi}{5} = \frac{\pi}{5}$

40. (a): Let $y = \sqrt{9-x^2} \;\Rightarrow\; y^2 = 9 - x^2$

$\Rightarrow\; x^2 = 9 - y^2 \;\Rightarrow\; x = \sqrt{9-y^2}$

Clearly, $9 - y^2 \geq 0 \Rightarrow y^2 \leq 9$

$\Rightarrow \quad -3 \leq y \leq 3$

But $y \geq 0$. Hence $0 \leq y \leq 3$.

41. (a): If A and B are independent

Then A' and B' are also independent

$\Rightarrow \quad P(A' \cap B') = P(A')P(B')$

$= (1 - P(A))(1 - P(B))$

42. (a): $e = \frac{\sqrt{a^2-b^2}}{a} = \frac{\sqrt{36-16}}{6} = \frac{2\sqrt{5}}{6}$

43. (b): Given, $|\vec{a}| = |\vec{b}| = 1$

Now, $|\sqrt{3}\vec{a} - \vec{b}|^2 = 3|\vec{a}|^2 + |\vec{b}|^2 - 2\sqrt{3}(\vec{a}\cdot\vec{b})$

$\Rightarrow \quad 1 = 3(1) + 1 - 2\sqrt{3}\,|\vec{a}||\vec{b}|\cos\theta$

$\Rightarrow \quad 1 = 4 - 2\sqrt{3}(1)(1)\cos\theta$

$\Rightarrow \quad \cos\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = 30°$

44. (c): For, $\vec{a}$ and $\vec{b}$ to be orthogonal $\vec{a}\cdot\vec{b} = 0$

$\Rightarrow \quad (2)(1) + (\lambda)(2) + (1)(3) = 0$

$\Rightarrow \quad 5 + 2\lambda = 0 \;\Rightarrow\; \lambda = -\frac{5}{2}$

45. (b): $\cos^2 45° - \sin^2 15° = \cos(45°+15°)\cos(45°-15°)$

$= \cos 60° \cos 30° = \frac{1}{2}\cdot\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}$

46. (b): Range of $\sec^{-1}x$ is $[0, \pi] - \left\{\frac{\pi}{2}\right\}$

47. (a): Here, $\vec{a}, \vec{b}, \vec{c}$ are unit vectors and $\vec{a} + \vec{b} + \vec{c} = \vec{0}$

$\therefore \quad (\vec{a} + \vec{b} + \vec{c})^2 = 0$

$\Rightarrow \quad |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = 0$

$\Rightarrow \quad 1 + 1 + 1 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = 0$

$\Rightarrow \quad \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a} = -\frac{3}{2}$

48. (c): Let $I = \int_{-\pi/2}^{\pi/2} \frac{dx}{e^{\sin x}+1}$ ...(i)

$\Rightarrow \quad I = \int_{-\pi/2}^{\pi/2} \frac{dx}{e^{-\sin x}+1} = \int_{-\pi/2}^{\pi/2} \frac{e^{\sin x}dx}{1+e^{\sin x}}$ ...(ii)

Adding (i) and (ii), we get

$$2I = \int_{-\pi/2}^{\pi/2} \frac{e^{\sin x}+1}{e^{\sin x}+1}\,dx$$

$$\Rightarrow \quad 2I = \int_{-\pi/2}^{\pi/2} 1dx = [x]_{-\pi/2}^{\pi/2} = \pi \Rightarrow I = \frac{\pi}{2}$$

49. (a) : $\dfrac{dV}{dS} = \dfrac{dV/dt}{dS/dt} = \dfrac{4\pi r^2 \frac{dr}{dt}}{8\pi r \frac{dr}{dt}} = \dfrac{r}{2} = \dfrac{4}{2} = 2\ \text{cm}^3/\text{cm}^2$

50 . (a) : $\displaystyle\int_0^{\pi/2} \frac{\tan^7 x}{\cot^7 x + \tan^7 x}\,dx = \frac{\pi}{4}$

$$\left[\because \int_0^{\pi/2} \frac{f(x)}{f(x)+f\left(\frac{\pi}{2}-x\right)}\,dx = \frac{\pi}{4}\right]$$

51. (c) : Here, $|x-2| \le 1 \Rightarrow -1 \le x-2 \le 1 \Rightarrow 1 \le x \le 3$

$\Rightarrow \quad x \in [1,3]$

52. (b): Here, $\displaystyle\int_{0.2}^{3.5} [x]dx = \int_{0.2}^{1} 0dx + \int_1^2 1dx + \int_2^3 2dx + \int_3^{3.5} 3dx$

$= 0 + [x]_1^2 + [2x]_2^3 + [3x]_3^{3.5} = 1 + 2(1) + 3(0.5) = 4.5$

53. (d): Given, area of triangle with vertices $(K, 0)$, $(4, 0)$, $(0, 2)$ is 4 sq. units *i.e.*,

$$\pm 4 = \frac{1}{2}\begin{vmatrix} K & 0 & 1 \\ 4 & 0 & 1 \\ 0 & 2 & 1 \end{vmatrix} \Rightarrow \frac{-2K+8}{2} = \pm 4$$

$\Rightarrow \quad K = 0$ or 8

54. (b): Given, $f(x)$ is continuous

$$\Rightarrow \quad \lim_{x\to 2^-} f(x) = \lim_{x \to 2^+} f(x) = f(2)$$

$$\Rightarrow \quad \lim_{x\to 2} (Kx^2) = 3 \ \Rightarrow \ 4K = 3 \ \Rightarrow \ K = \frac{3}{4}$$

55 . (None of the options is correct) :

If matrix B will be, $B = \dfrac{1}{\pi}\begin{bmatrix} -\cos^{-1}\pi x & \tan^{-1}\dfrac{x}{\pi} \\ \sin^{-1}\left(\dfrac{x}{\pi}\right) & -\tan^{-1}\pi x \end{bmatrix}$

Then, $A - B = \dfrac{1}{2}I$

56. (c) : $fog(x) = f(g(x)) = f(x^{1/3}) = 8(x^{1/3})^3 = 8x$

57. (b): We have, $\Delta = \begin{vmatrix} Ax & x^2 & 1 \\ By & y^2 & 1 \\ Cz & z^2 & 1 \end{vmatrix} = xyz\begin{vmatrix} A & x & \frac{1}{x} \\ B & y & \frac{1}{y} \\ C & z & \frac{1}{z} \end{vmatrix}$

$$= \begin{vmatrix} A & x & yz \\ B & y & xz \\ C & z & xy \end{vmatrix} = \begin{vmatrix} A & B & C \\ x & y & z \\ zy & zx & xy \end{vmatrix} = \Delta_1$$

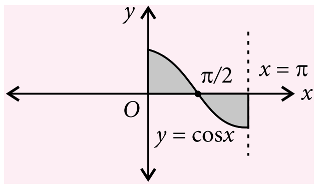
58. (a) : We have, $x = \sin^{-1}\dfrac{2t}{1+t^2} = 2\tan^{-1}t$

and $y = \tan^{-1}\dfrac{2t}{1-t^2} = 2\tan^{-1}t$

$\therefore \quad y = x \Rightarrow \dfrac{dy}{dx} = 1.$

59. (b): Reflection of the point (α,β,γ) in XY plane is $(\alpha,\beta,-\gamma)$.

60. (a) : Given, curves are $y = \cos x$; $x = 0$ and $x = \pi$



$\therefore \quad$ Required area $= 2\displaystyle\int_0^{\pi/2} \cos x dx$

$= 2[\sin x]_0^{\pi/2} = 2$ sq. units

◆◆

Solution Sender of Maths Musing

SET-173

1. V. Damodhar Reddy (Telangana)

SET-172

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MATHS MUSING

Maths Musing was started in January 2003 issue of Mathematics Today. The aim of Maths Musing is to augment the chances of bright students seeking admission into IITs with additional study material.

During the last 10 years there have been several changes in JEE pattern. To suit these changes Maths Musing also adopted the new pattern by changing the style of problems. Some of the Maths Musing problems have been adapted in JEE benefitting thousand of our readers. It is heartening that we receive solutions of Maths Musing problems from all over India.

Maths Musing has been receiving tremendous response from candidates preparing for JEE and teachers coaching them. We do hope that students will continue to use Maths Musing to boost up their ranks in JEE Main and Advanced.

PROBLEM Set 174

JEE MAIN

1. If $a_1 = 11, a_2 = 75, a_3 = 20, a_4 = 23$ and $a_n = a_{n-1} - a_{n-2} + a_{n-3} - a_{n-4}, n \geq 5$, then $a_3 - a_3 + a_3 =$
   (a) 53 (b) 58
   (c) 65 (d) 75

2. If $xyz = 1, x + \frac{1}{z} = 5, y + \frac{1}{x} = 29$, then $z + \frac{1}{y} =$
   (a) $\frac{1}{3}$ (b) $\frac{1}{4}$
   (c) 2 (d) $\frac{3}{4}$

3. In a triangle ABC, $\Delta\left(\frac{1}{r^2}+\frac{1}{r_1^2}+\frac{1}{r_2^2}+\frac{1}{r_3^2}\right) =$
   (a) $2\,\Sigma\tan A$ (b) $2\,\Sigma\cot A$
   (c) $4\,\Sigma\tan A$ (d) $4\,\Sigma\cot A$

4. A straight line through the point (a, b) meets x-axis at A and y-axis at B. O is the origin. If $(a, b) = (4, 1)$, then the minimum value of $OA + OB$ is
   (a) 7 (b) 8 (c) 9 (d) 10

5. If $x \neq a, y \neq b, z \neq c$ and $\begin{vmatrix} a & y & z \\ x & b & z \\ x & y & c \end{vmatrix} = 0$, then
   $\frac{x+a}{x-a} + \frac{y+b}{y-b} + \frac{z+c}{z-c} =$
   (a) -1 (b) 0
   (c) 1 (d) 2

JEE ADVANCED

6. A bag contains 30 tokens numbered serially from 0 to 30. The number of ways of selecting 3 tokens from the bag, so that the sum of the numbers on them is 30, is divisible by
   (a) 2 (b) 3 (c) 5 (d) 7

COMPREHENSION

Let $f(x) = [\sin^{-1} x] + [\cos^{-1} x],\ x \in [0,1]$

7. The number of points at which $f(x)$ is not differentiable is
   (a) 0 (b) 1 (c) 2 (d) 3

8. The solution set of the equation $f(x) = 0$ is an interval of length
   (a) $\sqrt{2}\sin\left(1-\frac{\pi}{4}\right)$ (b) $\sqrt{2}\sin\left(1+\frac{\pi}{4}\right)$
   (c) $\sqrt{2}\cos\left(1-\frac{\pi}{4}\right)$ (d) $\sqrt{2}\cos\left(1+\frac{\pi}{4}\right)$

INTEGER TYPE

9. Ten boys and two girls are to be seated in a row such that there are atleast 3 boys between the girls. The number of ways this can be done is $\lambda \cdot 12!$, where $\lambda =$

MATRIX MATCH

10. The curve $f(x, y) = 0$ lies in the first quadrant. The tangent at a point on it meets the positive x and y axes at A and B and O is the origin

	List-I		List-II
P.	$f(x, y) = 4xy - 1$	1.	$AB = 1$
Q.	$f(x, y) = x^2 + y^2 - 1$	2.	$OA + OB = 1$
R.	$f(x, y) = \sqrt{x} + \sqrt{y} - 1$	3.	$OA \cdot OB = 1$
S.	$f(x, y) = x^{2/3} + y^{2/3} - 1$	4.	$\frac{1}{OA^2} + \frac{1}{OB^2} = 1$

	P	Q	R	S
(a)	2	1	3	4
(b)	1	2	4	3
(c)	4	3	2	1
(d)	3	4	2	1

See Solution Set of Maths Musing 173 on page no. 85

20 CHALLENGING PROBLEMS

For Entrance Exams

1. Let x, y, z be real numbers such that $\cos x + \cos y + \cos z = 0$ and $\cos 3x + \cos 3y + \cos 3z = 0$, then $\cos 2x \cos 2y \cos 2z$

(a) ≤ 0
(b) ≥ 0
(c) depends on x, y, z values
(d) data insufficient

2. Eliminate θ from the system : $\lambda \cos 2\theta = \cos(\theta + \alpha)$ and $\lambda \sin 2\theta = 2\sin(\theta + \alpha)$

(a) $(\cos\alpha)^{2/3} - (\sin\alpha)^{2/3} = \lambda^{2/3}$
(b) $(\cos\alpha)^{2/3} + (\sin\alpha)^{2/3} = \lambda^{2/3}$
(c) $(\cos\alpha)^{1/3} - (\sin\alpha)^{1/3} = \lambda^{1/3}$
(d) $(\cos\alpha)^{1/3} - (\sin\alpha)^{1/3} = \lambda^{1/3}$

3. The bisector of $\angle BAC$ intersects the circumcircle of ΔABC at D. If $AB^2 + AC^2 = 2AD^2$ then angle of intersection of AD and BC is

(a) $30°$ (b) $45°$
(c) $60°$ (d) $90°$

4. The laws of points (x, y) satisfying the equations $x^2 + y\cos^2\alpha = x\sin\alpha\cos\alpha$ and $x\cos 2\alpha + y\sin 2\alpha = 0$ lies on (α is a parameter)

(a) circle (b) parabola
(c) ellipse (d) hyperbola

5. In ΔABC, $\angle ABC = \angle ACB = 40°$. If P is a point in the interior of the triangle such that $\angle PBC = 20°$ and $\angle PCB = 30°$ then

(a) $BP = BA$ (b) $BP = 2BA$
(c) $BP = \frac{1}{2}BA$ (d) $BP = 3BA$

6. Let ABC be a triangle of area $\frac{1}{2}$, then minimum value of $a^2 + \operatorname{cosec} A$ is

(a) $\sqrt{2}$ (b) $\sqrt{3}$ (c) $\sqrt{4}$ (d) $\sqrt{5}$

7. Let $\alpha, \beta, \gamma, \delta$ be positive numbers such that for all x, $\sin\alpha x + \sin\beta x = \sin\gamma x + \sin\delta x$, then $\gamma + \delta =$

(a) α (b) 2α (c) 3α (d) 4α

8. Let S be the set of all triangles ABC for which

$$5\left[\frac{1}{AP}+\frac{1}{BQ}+\frac{1}{CR}\right]-\frac{3}{\min.\{AP, BQ, CR\}}=\frac{6}{r},$$

where r is inradius and P, Q, R are points of tangency of incircle with sides AB, BC, CA respectively then all the triangles in the set S are

(a) scalene (b) isosceles
(c) equilateral (d) right angled

9. Let ABC be a triangle such that $\max.\{A, B\} = C + 30°$ and $\frac{R}{r} = \sqrt{3}+1$, R is circumradius, r is inradius then ΔABC is

(a) scalene (b) isosceles
(c) equilateral (d) right angled

10. Let $ABCDEFGHIJKL$ be a regular do-decagon then $\frac{AB}{AF}+\frac{AF}{AB}=$

(a) 1 (b) 2 (c) 3 (d) 4

11. Let $a, b, c, d \in [0, \pi]$ such that

$$2\cos a + 6\cos b + 7\cos c + 9\cos d = 0$$

and $2\sin a - 6\sin b + 7\sin c - 9\sin d = 0$ then

$$\frac{\cos(a+d)}{\cos(b+c)} =$$

(a) $\frac{7}{3}$ (b) $\frac{3}{7}$ (c) $\frac{3}{5}$ (d) $\frac{5}{3}$

12. If $\sin x\cos y + \sin y\cos z + \sin z\cos x = \frac{3}{2}$, then

(a) $\sin x = \cos 2y$ (b) $\sin x = \sin y$
(c) $\sin x = \cos y$ (d) $\sin x = \cos z$

By : Tapas Kr. Yogi, Mob : 9533632105.

13. In ΔABC, $\lambda = \sin A \sin B + \sin B \sin C + \sin C \sin A$ and $(1 + \sin A)(1 + \sin B)(1 + \sin C) = 2(\lambda + 1)$, then ΔABC is

(a) scalene (b) isosceles
(c) equilateral (d) right angled

14. Let ABC be a triangle such that $\sin^2 B + \sin^2 C = 1 + 2 \sin B \sin C \cos A$, then ΔABC is

(a) scalene (b) isosceles
(c) equilateral (d) right angled

15. In ΔABC, $\left(\cot \frac{A}{2}\right)^2 + 4\cot^2\frac{B}{2} + 9\cot^2\frac{C}{2} = \left(\frac{6s}{7r}\right)^2$ where $s \to$ semiperimeter and $r \to$ inradius, then the ratio of the sides of the triangle is

(a) 45 : 40 : 13 (b) 50 : 55 : 45
(c) 55 : 65 : 70 (d) 60 : 65 : 80

16. Let n be a positive integer and for real numbers λ and a_{kl}. $(k, l = 1, 2, 3, \ldots n)$ $(k > l)$, we have

$$\frac{\sin^2 nx}{\sin^2 x} = \lambda + \sum_{1 \le l < k \le n} a_{kl} \cos(2k - 2l)x$$

$(x \ne m\pi, m \in I)$ then $\lambda =$

(a) l (b) k (c) n (d) $k - l$

17. In ΔABC, $\angle A = 30°$ and an inscribed circle of fixed radius r is drawn. If ΔABC has least perimeter then $\angle B =$

(a) 30° (b) 45° (c) 60° (d) 75°

18. If A_1, A_2, A_3, A_4 are the angles of a convex quad. then max. value of

$$\sin\left(\frac{A_1}{2}\right) + \sin\left(\frac{A_2}{2}\right) + \sin\left(\frac{A_3}{2}\right) + \sin\left(\frac{A_4}{2}\right) \text{ is}$$

(a) $\sqrt{2}$ (b) $2\sqrt{2}$ (c) $3\sqrt{2}$ (d) $4\sqrt{2}$

19. If a, b, c, k are constants and α, β, γ are variables subject to the relation $a \tan\alpha + b\tan\beta + c\tan\gamma = k$ then minimum value of $\tan^2\alpha + \tan^2\beta + \tan^2\gamma$ is

(a) $\frac{k^2}{(a+b+c)^2}$ (b) $\frac{k^2}{a^2+b^2+c^2}$

(c) $\frac{(a+b+c)^2}{k^2}$ (d) $\frac{a^2+b^2+c^2}{k^2}$

20. For $x \in R$, the minimum value of $|\sin x + \cos x + \tan x + \sec x + \text{cosec } x + \cot x|$ is

(a) $2\sqrt{2}$ (b) $2\sqrt{2} - 1$
(c) $2\sqrt{2} + 1$ (d) $\sqrt{2} - 1$

SOLUTIONS

1. **(a)** : Using $\cos 3x = 4\cos^3 x - 3\cos x$.

We have, $\sum \cos^3 x = 0$

using identity $\sum a^3 - 3abc = (\sum a)(\sum a^2 - \sum ab)$

We have, $abc = \cos x \cos y \cos z = 0$

Let $\cos z = 0$. So, $\cos x = -\cos y$ and

$\cos 2x = \cos 2y$ and $\cos 2z = -1$

So, $\cos 2x \cos 2y \cos 2z = -\cos^2 2x \le 0$.

2. **(b)** : Expanding the given equations, we have

$(\cos\theta\cos\alpha - \sin\theta\sin\alpha) = \lambda(\cos^2\theta - \sin^2\theta)$

and $\sin\theta\cos\alpha + \cos\theta\sin\alpha = \lambda\sin\theta\cos\theta$

$\Rightarrow \cos\alpha = \lambda\cos^3\theta$ and $\sin\alpha = \lambda\sin^3\theta$

Hence, $(\cos\alpha)^{2/3} + (\sin\alpha)^{2/3} = \lambda^{2/3}$.

3. **(b)** : Let $\angle CAB = \alpha$, $\angle ABC = \beta$, $\angle BCA = \gamma$ then using sine rule, we have

$$\frac{AB}{\sin\gamma} = \frac{AC}{\sin\beta} = \frac{AD}{\sin(\alpha/2 + \beta)}$$

So, $AB^2 + AC^2 = 2AD^2$ becomes

$\sin^2\gamma + \sin^2\beta = 2\sin^2(\alpha/2 + \beta)$

Simplifying, $\cos(\beta - \gamma)[1 + \cos(\beta + \gamma)] = 0$

i.e., $\cos(\beta - \gamma) = 0$. So, $\alpha + 2\beta = \pi/2$

So, $\angle AEC = \frac{\alpha}{2} + \beta = \frac{\pi}{4} = 45°$

4. **(b)** : Simplifying the first equation, we have

$x\sin 2\alpha - y\cos 2\alpha = 2x^2 + y$

and the other given equation is

$x\cos 2\alpha + y\sin 2\alpha = 0$

Solving, we have

$$\sin 2\alpha = \frac{x(2x^2 + y)}{x^2 + y^2} \text{ and } \cos 2\alpha = \frac{-y(2x^2 + y)}{x^2 + y^2}$$

Hence, $\sin^2 2\alpha + \cos^2 2\alpha = 1$ gives

$\frac{(2x^2 + y)^2}{x^2 + y^2} = 1$ *i.e.*, $4x^2 + 4y - 1 = 0$, Parabola.

5. **(a)** : Let us assume that $AB = AC = 1$ unit then

$BC = 2\cos 40°$ and $\angle BPC = 130°$

Applying sine rule in ΔBPC, we have

$$\frac{BP}{\sin(30°)} = \frac{BC}{\sin(130°)}$$

$$\Rightarrow BP = \frac{BC \cdot \sin 30°}{\cos 40°} = \frac{2\cos 40° \cdot \sin 30°}{\cos 40°} = 1$$

i.e., $BP = AB$.

6. (d) : Given, area = $\frac{1}{2} \Rightarrow \frac{1}{2}bc\sin A = \frac{1}{2}$

$\Rightarrow$ cosec $A = bc$ and $[bc \geq 1]$

Now,

$a^2 + \text{cosec } A = a^2 + bc = b^2 + c^2 - 2bc\cos A + bc$ [cosine rule]

$$= b^2 + c^2 - 2bc\sqrt{1-\sin^2 A} + bc$$

$$= b^2 + c^2 - 2\sqrt{b^2c^2 - (bc\sin A)^2} + bc$$

$$= b^2 + bc + c^2 - 2\sqrt{b^2c^2 - 1}$$

$$\geq 3bc - 2\sqrt{b^2c^2 - 1}$$

Let $x = bc\ (\geq 1)$ then $y = 3x - 2\sqrt{x^2-1}$ gives $5x^2 - 6xy + y^2 + 4 = 0$. As $x \in R$, $D \geq 0$ gives $(-6y)^2 - 20(y^2 + 4) \geq 0$ *i.e.*, $y \geq \sqrt{5}$.

7. (b) : Differentiating the given identity three times, we have, $\alpha^3\cos\alpha x + \beta^3\cos\beta x = \gamma^3\cos\gamma x + \delta^3\cos\delta x$

Also, $\alpha\cos\alpha x + \beta\cos\beta x = \gamma\cos\gamma x + \delta\cos\delta x$

In particular for $x = 0$, we have

$\alpha + \beta = \gamma + \delta$ and $\alpha^3 + \beta^3 = \gamma^3 + \delta^3$

i.e., $(\alpha + \beta)^3 = (\gamma + \delta)^3$

$\Rightarrow \alpha\beta = \gamma\delta$ on simplification.

So, $(\alpha - \gamma)(\alpha - \delta) = \alpha^2 - \alpha(\gamma + \delta) + \gamma\delta$
$= \alpha^2 - \alpha(\alpha + \beta) + \alpha\beta = 0$

$\Rightarrow \gamma = \alpha$ and $\delta = \alpha$ *i.e.*, $\gamma + \delta = 2\alpha$

8. (b) : Let us assume that min. $\{AP, BQ, CR\} = AP$ and let $\tan(A/2) = x$, $\tan(B/2) = y$, $\tan(C/2) = z$.

So, $AP = \frac{r}{x}$, $BQ = \frac{r}{y}$ and $CR = \frac{r}{z}$

Now, the relation in question becomes

$2x + 5y + 5z = 6$

and in any Δ, we know that $xy + yz + zx = 1$.

Now, eliminating (x) from these two equations, we have $5y^2 + 5z^2 + 8yz - 6y - 6z + 2 = 0$

i.e., $(3y - 1)^2 + (3z - 1)^2 = 4(y - z)^2$

or, $5\alpha^2 + 5\beta^2 + 8\alpha\beta = 0$ [where $3y - 1 = \alpha$, $3z - 1 = \beta$]

$\Rightarrow \alpha = 0 = \beta$ for real solutions.

i.e., $y = z = \frac{1}{3}$ and so, $x = \frac{4}{3}$

i.e., isosceles triangle.

9. (d) : Let max. $\{A, B\} = A$ then A.T.Q, $A - C = 30°$ and using the identity

$$r = 4R\sin\left(\frac{A}{2}\right).\sin\left(\frac{B}{2}\right).\sin\left(\frac{C}{2}\right)$$

We have, $r = 4r\sqrt{3+1}\left[\sin\left(\frac{A}{2}\right)\cdot\sin\left(\frac{B}{2}\right)\cdot\sin\left(\frac{C}{2}\right)\right]$

i.e., $\frac{\sqrt{3}-1}{4} = \sin\frac{B}{2}\left[\cos\frac{A-C}{2} - \cos\frac{A+C}{2}\right]$

i.e., $\frac{\sqrt{3}-1}{4} = \sin\frac{B}{2}\left[\cos\left(\frac{30°}{2}\right) - \cos\left(\frac{180° - B}{2}\right)\right]$

i.e., $\sin^2\frac{B}{2} - \sin\left(\frac{B}{2}\right).\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right) + \left(\frac{\sqrt{3}-1}{4}\right) = 0$

Solving, $\sin(B/2) = \frac{\sqrt{6}-\sqrt{2}}{4}$ or $\frac{\sqrt{2}}{2}$

i.e., $\frac{B}{2} = 15°$ or $45°$.

But $B = 90°$ is not possible. Hence, $B = 30°$ and $A = 90°$, $C = 60°$, *i.e.*, [Right angled Δ]

10. (d) : Let R be the circumradius then

$AB = 2R\sin\left(\frac{\pi}{12}\right)$ and $AF = 2R\sin\left(\frac{5\pi}{12}\right)$

So, the required quantity is $\frac{2R\sin\theta}{2R\sin 5\theta} + \frac{2R\sin 5\theta}{2R\sin\theta}$ $\left[\text{where } \theta = \frac{\pi}{12}\right]$

$$= \frac{\sin^2\theta + \sin^2 5\theta}{\sin 5\theta\sin\theta} = \frac{1 - \cos 2\theta + 1 - \cos 10\theta}{\cos 4\theta - \cos 6\theta}$$

= 4 (on simplification)

11. (a) : Rearranging the two equations, we have

$2\sin a - 9\sin d = 6\sin b - 7\sin c$

and $2\cos a + 9\cos d = -6\cos b - 7\cos c$

Squaring and adding both the equations, we have

$85 + 36\cos(a + d) = 85 + 84\cos(b + c)$

i.e., $\frac{\cos(a+d)}{\cos(b+c)} = \frac{84}{36} = \frac{7}{3}$

12. (c): The given equation can be rewritten as,

$(\sin x - \cos y)^2 + (\sin y - \cos z)^2 + (\sin z - \cos x)^2 = 0$

i.e., $\sin x = \cos y$

13. (d) : Equating the two λ values and simplifying, we have

$(1 - \sin A)(1 - \sin B)(1 - \sin C) = 0$

i.e., $\sin A = 1$ or $\sin B = 1$ or $\sin C = 1$

i.e., ABC is right angled triangle.

14. (d) : Using cosine rule, we have

$$a^2 + 2bc \cos A = b^2 + c^2$$

And using sine rule, here, we have

$$\sin^2 B + \sin^2 C = \sin^2 A + 2 \sin B \sin C \cos A$$

Now comparing this with equation given in question, we have, $\sin^2 A = 1$ *i.e.*, $A = 90°$

15. (a) : In any triangle, we have

$$\cot\frac{A}{2}+\cot\frac{B}{2}+\cot\frac{C}{2}=\cot\frac{A}{2}\cdot\cot\frac{B}{2}\cdot\cot\frac{C}{2}=\frac{s}{r}$$

So, the given relation in question becomes,

$$(6^2+3^2+2^2)\left[\left(\cot\frac{A}{2}\right)^2+\left(2\cot\frac{B}{2}\right)^2+\left(3\cot\frac{C}{2}\right)^2\right]$$

$$=\left(6\cot\frac{A}{2}+6\cot\frac{B}{2}+6\cot\frac{C}{2}\right)^2$$

i.e., Equality in Cauchy-Schwarz inequality.

So, $\dfrac{\cot(A/2)}{6}=\dfrac{2\cot(B/2)}{3}=\dfrac{3\cot(C/2)}{2}$

i.e., $\cot(A/2)=7,\ \cot(B/2)=\dfrac{7}{4}$ and $\cot(C/2)=\dfrac{7}{9}$

$\Rightarrow \sin A=\dfrac{7}{25},\ \ \sin B=\dfrac{56}{65},\ \sin C=\dfrac{126}{130}$

i.e., Sides of the triangle are 26, 80 and 90.

16. (c): Using the two identities, we have

$$S_1 = \sum_{m=1}^{n}\cos(2mx)=\frac{\sin nx\cdot\cos(n+1)x}{\sin x}$$

and $S_2 = \displaystyle\sum_{m=1}^{n}\sin(2mx)=\frac{\sin nx\cdot\sin(n+1)x}{\sin x}$

$$S_1^2+S_2^2 = \left(\frac{\sin nx}{\sin x}\right)^2$$

But $S_1^2+S_2^2 = (\cos 2x + \cos 4x + \ldots + \cos nx)^2 + (\sin 2x + \sin 4x + \ldots + \sin nx)^2$

$$= n+2\sum(\cos 2kx\cos 2lx+\sin 2kx\sin 2lx)$$
$$1 \le l < k \le n$$

i.e., $S_1^2+S_2^2 = n+\displaystyle\sum_{1\le l<k\le n}\cos 2(k-l)x$

i.e., $\lambda = n$

17. (d) : In any triangle, we have $\sum\cot(A/2)=\dfrac{s}{r}$

So, perimeter $2s = 2r\,.\sum\cot(A/2)$

Since A, r are fixed, s is min. when $\sum\cot(A/2)$ is min.

i.e., $\cot(B/2) + \cot(C/2)$ is min.

i.e., $\dfrac{\cos(A/2)}{\sin(B/2)\sin(C/2)}$ is min.

or, $\sin(B/2)\cdot\sin(C/2)$ is max.

i.e., $\dfrac{1}{2}\left[\cos\left(\dfrac{B-C}{2}\right)-\sin\left(\dfrac{A}{2}\right)\right]$ is max.

i.e., $\cos\left(\dfrac{B-C}{2}\right) = 1$ for max. *i.e.*, $B = C$.

$\therefore$ ΔABC is an isosceles Δ with $\angle B = \angle C = 75°$.

18. (b) : $A_1 + A_2 + A_3 + A_4 = 2\pi$,

So, $\dfrac{A_1}{2}+\dfrac{A_2}{2}+\dfrac{A_3}{2}+\dfrac{A_4}{2}=\pi$

Now, $f(x) = \sin x$ is a concave function.

So, $\sin\left(\dfrac{A_1}{2}\right)+\sin\left(\dfrac{A_2}{2}\right)+\sin\left(\dfrac{A_3}{2}\right)+\sin\left(\dfrac{A_4}{2}\right)$ is max.

When $\dfrac{A_1}{2}=\dfrac{A_2}{2}=\dfrac{A_3}{2}=\dfrac{A_4}{2}$ = each = $\dfrac{\pi}{4}$

[Result of Jensen's inequality]

So, required max. value is $4\cdot\sin(\pi/4) = 2\sqrt{2}$.

19. (b) : Use the identity

$$\sum(b\tan\gamma-c\tan\beta)^2 = (a^2+b^2+c^2)(\tan^2\alpha+\tan^2\beta+\tan^2\gamma) - (a\tan\alpha+b\tan\beta+c\tan\gamma)^2$$

We have, RHS ≥ 0

$\Rightarrow$ $(\sum a^2)\cdot(\sum\tan^2\alpha)-k^2\ge 0$

i.e., $\left(\sum\tan^2\alpha\right)_{\min}=\dfrac{k^2}{\Sigma a^2}$

20. (b) : Let $E = |\sin x + \cos x + \tan x + \sec x + \operatorname{cosec} x + \cot x|$

Putting $a = \sin x$, $b = \cos x$, $c = a + b$

We have on simplification, the given expression

$$E=\left|c+\frac{2}{c-1}\right|=\left|c-1+\frac{2}{c-1}+1\right|$$

where $c\in[-\sqrt{2},\sqrt{2}]$

Using AM $\ge$ GM on $(c-1)$ and $\left(\dfrac{2}{c-1}\right)$, we have

$E_{\min} = |-2\sqrt{2}+1|$ *i.e.*, $2\sqrt{2}-1$

MATH archives

Math Archives, as the title itself suggests, is a collection of various challenging problems related to the topics of IIT-JEE Syllabus. This section is basically aimed at providing an extra insight and knowledge to the candidates preparing for IIT-JEE. In every issue of MT, challenging problems are offered with detailed solution. The readers' comments and suggestions regarding the problems and solutions offered are always welcome.

1. The sum of all the non-real roots of $(x^2 + x - 2)(x^2 + x - 3) = 12$ is

(a) 1 (b) −1
(c) 6 (d) −2

2. Statement-1 : The equation $\sin x + x\cos x = 0$ has at least one root in the interval $(0, \pi)$.
Statement-2 : Between any two roots of $f(x) = 0$, there exists atleast one root of $f'(x) = 0$.

(a) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is true; Statement-2 is not correct explanation for Statement-1.
(c) Statement-1 is true, Statement-2 is false.
(d) Statement-1 is false, Statement-2 is true.

3. If LCM of p, q is $r^2t^4s^2$, where r, s, t are prime numbers and p, q are positive integers . Then the number of ordered pairs (p, q) is

(a) 252 (b) 254
(c) 225 (d) 224

4. Let $f(x) = \max\{x, x^3\}$ $x \in R$ the set of points where $f(x)$ is not differentiable is

(a) $\{-1, 1\}$ (b) $\{-1, 0\}$
(c) $\{0, 1\}$ (d) $\{-1, 0, 1\}$

5. Statement-1: The sum of the first 30 terms of the sequence 1, 2, 4, 7, 11, 16, 22, 29, 37, 46 ... is 4520.
Statement-2: The successive differences of the terms of the sequence form an A.P.

(a) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is true; Statement-2 is not correct explanation for Statement-1.
(c) Statement-1 is true, Statement-2 is false.
(d) Statement-1 is false, Statement-2 is true.

6. $\displaystyle\sum_{r=1}^{n} \tan^{-1}\left(\frac{1}{2r^2}\right) =$

(a) $\tan^{-1} n$ (b) $\tan^{-1}\dfrac{n}{n+1}$
(c) $\tan^{-1}\dfrac{n}{n+2}$ (d) $\tan^{-1}\left(\dfrac{n+1}{n+2}\right)$

7. The statement $p \to (q \to p)$ is equivalent to

(a) $p \to (p \to q)$ (b) $p \to (p \vee q)$
(c) $p \to (p \wedge q)$ (d) $p \to (p \leftrightarrow q)$

8. The remainder left out when $8^{2n} - (62)^{2n+1}$ is divisible by 9 (where $n \in N$)

(a) 0 (b) 2 (c) 7 (d) 8

9. Consider all functions that can be defined from the set $A = \{1, 2, 3\}$ to the set $B = \{1, 2, 3, 4, 5\}$. A function $f(x)$ is selected at random from these functions. The probability that, selected function satisfies $f(i) \le f(j)$ for $i < j$, is equal to

(a) $\dfrac{6}{25}$ (b) $\dfrac{12}{25}$
(c) $\dfrac{2}{5}$ (d) none of these

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10. $\int_0^{\pi}[\cot x]dx =$

(where $[\cdot]$ denotes the greatest integer function)

(a) $\frac{\pi}{2}$ (b) 1 (c) -1 (d) $-\frac{\pi}{2}$

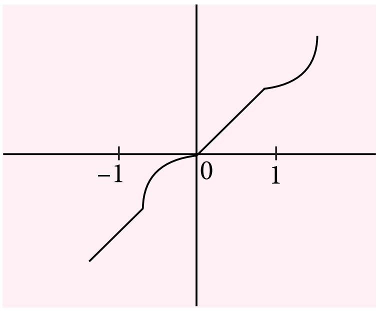
SOLUTIONS

1. **(b) :** Put $x^2 + x = y$ then, we have $y^2 - 5y - 6 = 0$
$\Rightarrow (y - 6)(y + 1) = 0$
$\Rightarrow x^2 + x - 6 = 0$ or $x^2 + x + 1 = 0$
$\Rightarrow x = -3, 2$ or $x = \omega, \omega^2$
Sum of non-real roots $= \omega + \omega^2 = -1$

2. **(c) :** Take $f(x) = x\sin x$, which is continuous in $[0, \pi]$ and differentiable in $(0, \pi)$. Also $f(0) = f(\pi) = 0$
By Rolle's theorem, there exists at least one root of $f'(x) = 0 \Rightarrow x\cos x + \sin x = 0$

3. **(c) :** No. of ordered pairs
$= (2(2) + 1)(2(4) + 1)(2(2) + 1)$
$= 5 \times 9 \times 5 = 225$

4. **(d) :**



5. **(d) :** $a_2 - a_1 = 1,\ a_3 - a_2 = 2,\ a_4 - a_3 = 3, \ldots$

$$a_n = 1 + \frac{n(n-1)}{2} = \frac{n^2 - n + 2}{2}$$

$$\therefore \quad \text{Sum} = \frac{1}{2}\left\{\frac{30\cdot 31\cdot 61}{6} - \frac{30\cdot 31}{2} + 2\cdot 30\right\} = 4525$$

6. **(b) :** $\frac{1}{2r^2} = \frac{2}{4r^2} = \frac{(2r+1)-(2r-1)}{1+(4r^2-1)}$

Sum $= \Sigma(\tan^{-1}(2r + 1) - \tan^{-1}(2r - 1))$

$= \tan^{-1}(2n + 1) - \tan^{-1}(1) = \tan^{-1}\left(\frac{n}{n+1}\right)$

7. **(b) :** $p \to (q \to p) \equiv \sim p \vee (q \to p)$
$\equiv \sim p \vee (\sim q \vee p) \equiv p \to (p \vee q)$

8. **(b) :** $(1 + 63)^n - (63 - 1)^{2n+1}$
Remainder is 2.

9. **(d) :** Total Function $= 5^3$

10. **(d) :** Let $\ell = \int_0^{\pi}[\cot x]dx \Rightarrow \ell = \int_0^{\pi}[\cot(\pi - x)]dx$

$$\therefore \quad \ell + \ell = \int_0^{\pi}(-1)dx \left[\because [x] + [-x] = \begin{cases} -1 \text{ if } x \notin z \\ 0 \text{ if } x \in z \end{cases}\right]$$

$$\therefore \quad \ell = -\frac{\pi}{2}$$

◆◆

OLYMPIAD CORNER

1. $a_1, \ldots, a_k, a_{k+1}, \ldots, a_n$ are positive numbers ($k < n$). Suppose that the values of $a_{k+1}, \ldots, a_n$ are fixed. How should one choose the values of $a_1, \ldots, a_n$ in order to minimize $\sum_{i,j,i \neq j} \frac{a_i}{a_j}$?

2. Let m be a positive integer. Define the sequence $a_0, a_1, a_2, \ldots$ by $a_0 = 0$, $a_1 = m$ and $a_{n+1} = m^2 a_n - a_{n-1}$ for $n = 1, 2, 3, \ldots$. Prove that an ordered pair (a, b) of non-negative integers, with $a \leq b$, gives a solution to the equation
$$\frac{a^2 + b^2}{ab + 1} = m^2$$
if and only if (a, b) is of the form (a_n, a_{n+1}) for some $n \geq 0$.

3. In a ΔABC, $\angle C = 2\angle B$. P is a point in the interior of ΔABC satisfying that $AP = AC$ and $PB = PC$. Show that AP trisects $\angle A$.

4. Determine all the possible values of the sum of the digits of the perfect squares.

5. $ABCD$ is a convex quadrilateral and O is the intersection of its diagonals. Let L, M, N be the mid-points of DB, BC, CA respectively. Suppose that AL, OM, DN are concurrent. Show that either $AD \parallel BC$ or $[ABCD] = 2[OBC]$.

SOLUTIONS

1. To minimize the given rational function, choose
$$a_i = \left(\frac{a_{k+1} + \ldots + a_n}{\frac{1}{a_{k+1}} + \ldots + \frac{1}{a_n}} \right)^{1/2} = (\mathrm{A} \cdot \mathrm{H})^{1/2}, i = 1, 2, \ldots, k$$
where A is the arithmetic mean and H is the harmonic mean of $a_{k+1}, \ldots, a_n$.

To prove this, we will be forgiven if we change notation : let $x_i = a_i$, $i = 1, 2, \ldots, k$ and $b_r = a_{k+r}$, $r = 1, \ldots, m$ with $k + m = n$ and denote the given rational function $F(x_1, \ldots, x_k)$.

Then we have $F(x_1, \ldots, x_k) = X + Y + B$, where
$$X = \sum_{1 \leq i < j \leq k} \left(\frac{x_i}{x_j} + \frac{x_j}{x_i} \right),$$
$$Y = \sum_{1 \leq i \leq k} \sum_{1 \leq r \leq m} \left(\frac{x_i}{b_r} + \frac{b_r}{x_i} \right),$$
$$B = \sum_{1 \leq r < s \leq m} \left(\frac{b_r}{b_s} + \frac{b_s}{b_r} \right).$$
Note that B is fixed and Y can be improved to
$$Y = \sum_{1 \leq i \leq k} \left(\left(\sum_{1 \leq r \leq m} \frac{1}{b_r} \right) x_i + \left(\sum_{1 \leq r \leq m} b_i \right) \frac{1}{x_i} \right)$$
$$= \sum_i \left(\frac{m}{\mathrm{H}} x_i + \frac{m\mathrm{A}}{x_i} \right)$$
where A is the arithmetic mean and H is the harmonic mean of the b_r.

Now we recall that the simple function $\alpha x + \frac{\beta}{x}$ (with α, β, x all positive) assumes its minimum when $\alpha x = \frac{\beta}{x}$; that is $x = \sqrt{\beta / \alpha}$. Thus each of the terms in Y (and so Y itself) assumes its minimum when we choose, for $i = 1, 2, \ldots, k$,
$x_i = \sqrt{\frac{m\mathrm{A}}{(m/\mathrm{H})}} = \sqrt{\mathrm{AH}}$, as asserted.

But there is more. It is also known that each term in X, (and so X itself) assumes its minimum when $x_i = x_j$, with $1 \leq i < j \leq k$. Thus choosing all $x_i = \sqrt{\mathrm{AH}}$ minimizes both X and Y and, since B

is fixed, minimizes $F(x_1, ..., x_k)$ as claimed.

2. Let us first prove by induction that

$$\frac{a_n^2 + a_{n+1}^2}{a_n \cdot a_{n+1} + 1} = m^2 \text{ for all } n \geq 0.$$

Proof : Base case $(n = 0)$: $\frac{a_0^2 + a_1^2}{a_0 \cdot a_1 + 1} = \frac{0 + m^2}{0 + 1} = m^2$.

Now, let us assume that it is true for $n = k$, $k \geq 0$.

Then, $\frac{a_k^2 + a_{k+1}^2}{a_k \cdot a_{k+1} + 1} = m^2$

$$a_k^2 + a_{k+1}^2 = m^2 \cdot a_k \cdot a_{k+1} + m^2$$

$$a_{k+1}^2 + m^4 a_{k+1}^2 - 2m^2 \cdot a_k \cdot a_{k+1} + a_k^2 = m^2 + m^4 a_{k+1}^2 - m^2 \cdot a_k \cdot a_{k+1}$$

$$a_{k+1}^2 + (m^2 a_{k+1} - a_k)^2 = m^2 + m^2 a_{k+1}(m^2 a_{k+1} - a_k)$$

$$a_{k+1}^2 + a_{k+2}^2 = m^2 + m^2 \cdot a_{k+1} \cdot a_{k+2}$$

Therefore, $\frac{a_{k+1}^2 + a_{k+2}^2}{a_{k+1} \cdot a_{k+2} + 1} = m^2$, proving the induction. Hence (a_n, a_{n+1}) is a solution to $\frac{a^2 + b^2}{ab + 1} = m^2$ for all $n \geq 0$.

Now, consider the equation $\frac{a^2 + b^2}{ab + 1} = m^2$ and suppose $(a, b) = (x, y)$ is a solution with $0 \leq x \leq y$. Then

$$\frac{x^2 + y^2}{xy + 1} = m^2 \quad \text{...(1)}$$

If $x = 0$ then it is easily seen that $y = m$, so $(x, y) = (a_0, a_1)$. Since we are given $x \geq 0$, suppose now that $x > 0$.

Let us show that $y \leq m^2 x$.

Proof by contradiction : Assume that $y > m^2 x$. Then $y = m^2 x + k$ where $k \geq 1$.

Substituting into (1) we get

$$\frac{x^2 + (m^2 x + k)^2}{(x)(m^2 x + k) + 1} = m^2$$

$$x^2 + m^4 x^2 + 2m^2 xk + k^2 = m^4 x^2 + m^2 kx + m^2$$

$$(x^2 + k^2) + m^2(kx - 1) = 0.$$

Now, $m^2(kx - 1) \geq 0$ since $kx \geq 1$ and $x^2 + k^2 \geq x^2 + 1 \geq 1$ so $(x^2 + k^2) + m^2(kx - 1) \neq 0$.

Thus we have a contradiction, so $y \leq m^2 x$ if $x > 0$.

Now substitute $y = m^2 x - x_1$, where $0 \leq x_1 < m^2 x$, into (1).

We have

$$\frac{x^2 + (m^2 x - x_1)^2}{x(m^2 x - x_1) + 1} = m^2$$

$$x^2 + m^4 x^2 - 2m^2 x \cdot x_1 + x_1^2 = m^4 x^2 - m^2 x \cdot x_1 + m^2$$

$$x^2 + x_1^2 = m^2(x \cdot x_1 + 1)$$

$$\frac{x^2 + x_1^2}{x \cdot x_1 + 1} = m^2 \quad \text{....(2)}$$

If $x_1 = 0$, then $x^2 = m^2$. Hence $x = m$ and $(x_1, x) = (0, m) = (a_0, a_1)$. But $y = m^2 x - x_1 = a_2$, so $(x, y) = (a_1, a_2)$.

Thus suppose $x_1 > 0$.

Let us now show that $x_1 < x$.

Proof by contradiction: Assume $x_1 \geq x$.

Then $m^2 x - y \geq x$, since $y = m^2 x - x_1$, and $\left(\frac{x^2 + y^2}{xy + 1}\right)x - y \geq x$, since (x, y) is a solution to $\frac{a^2 + b^2}{ab + 1} = m^2$.

So $x^3 + xy^2 \geq x^2 y + xy^2 + x + y$.

Hence $x^3 \geq x^2 y + x + y$, which is a contradiction since $y \geq x > 0$.

With the same proof that $y \leq m^2 x$, we have $x \leq m^2 x_1$. So the substitution $x = m^2 x_1 - x_2$ with $x_2 \geq 0$ is valid.

Substituting $x = m^2 x_1 - x_2$ into (2) gives $\frac{x_1^2 + x_2^2}{x_1 \cdot x_2 + 1} = m^2$.

If $x_2 \neq 0$, then we continue with the substitution $x_i = m^2_{x_{i+1}} - x_{i+2}$ (*) until we get $\frac{x_j^2 + x_{j+1}^2}{x_j \cdot x_{j+1} + 1} = m^2$ and $x_{j+1} = 0$. (The sequence x_i is decreasing, non-negative and integer.)

So, if $x_{j+1} = 0$, then $x_j^2 = m^2$ so $x_j = m$ and $(x_{j+1}, x_j) = (0, m) = (a_0, a_1)$.

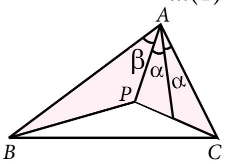
Then $(x_j, x_{j-1}) = (a_1, a_2)$ since $x_{j-1} = m^2 x_j - x_{j+1}$ (from (*)).

Continuing, we have $(x_1, x) = (a_{n-1}, a_n)$ for some n. Then $(x, y) = (a_n, a_{n+1})$.

Hence $\frac{a^2 + b^2}{ab + 1} = m^2$ has solutions (a, b) if and only if $(a, b) = (a_n, a_{n+1})$ for some n.

3. Let $\angle PAC$ and $\angle BAP$ be 2α and β respectively. Then, since $\angle C = 2\angle B$, we deduce from $A + B + C = 180°$ that

$2\alpha + \beta + 3B = 180°$. ...(1)



The angles at the base of the isosceles triangle PAC are each $90° - \alpha$. Also ΔBPC is isosceles, having base angles $C - (90° - \alpha) = 2B + \alpha - 90°$, and so $\angle BPA = 180° - (\angle PBA + \angle BAP)$

$= 180° - [B - (2B + \alpha - 90°) + 180° - 2\alpha - 3B]$

$= 4B + 3\alpha - 90°$

As usual, let a, b and c denote the lengths of the sides BC, AC and AB. By the Law of Cosines, applied to ΔBPA, where $\overline{PA} = b$ and $\overline{PB} = \overline{PC} = 2b \sin\alpha$,

$c^2 = b^2 + (2b \sin\alpha)^2 - 2{\cdot}b{\cdot}2b \sin\alpha{\cdot}\cos(4B + 3\alpha - 90°)$,

so that

$c^2 = b^2 [1 + 4\sin^2\alpha - 4\sin\alpha \sin(4B + 3\alpha)]$ ...(2)

We now use the fact that $\angle C = 2\angle B$ is equivalent to the condition $c^2 = b(b + a)$.

Since $a = 2\cdot \overline{PC} \cdot \cos(2B + \alpha - 90°)$

$= 4b \sin\alpha \sin(2B + \alpha)$, we have

$c^2 = b^2 [1 + 4\sin\alpha \sin(2B + \alpha)]$ ...(3)

Therefore, from (2) and (3), we get

$b^2 [1 + 4\sin^2\alpha - 4\sin\alpha \sin(4B + 3\alpha)]$
$= b^2 [1 + 4\sin\alpha \sin(2B + \alpha)]$,

which simplifies to

$\sin\alpha - \sin(4B + 3\alpha) = \sin(2B + \alpha)$.

Since $\sin\alpha - \sin(4B + 3\alpha) = -2\cos(2B + 2\alpha)\sin(2B + \alpha)$, this equation may be rewritten as $\sin(2B + \alpha).\ [1 + 2\cos(2B + 2\alpha)] = 0$

Since, from (1), $2B + \alpha < 180°$, we must have $1 + 2\cos(2B + 2\alpha) = 0$, giving $\cos(2B + 2\alpha) = -1/2$; that is,

$2B + 2\alpha = 120°$ ...(4)

Since, again from (1), $2B + 2\alpha < 180°$

Finally, we may eliminate B between (1) and (4) to obtain $\alpha = \beta$. The result follows.

4. The squares can only be 0, 1, 4 or 7 mod 9.

Thus the sum of the digits of a perfect square cannot be 2, 3, 5, 6 or 8 mod 9, since the number itself would then be 2, 3, 5, 6 or 8 mod 9.

We shall show that the sum of the digits of a perfect square can take every value of the form 0, 1, 4 or 7 mod 9.

$(10^m - 1)^2 = 10^{2m} - 2\cdot 10^m + 1$

$$= \underbrace{99\ldots9}_{m-1}8\underbrace{0\ldots0}_{m-1}1,\ m \ge 1.$$

The sum of the digits is $9m$, giving all the values greater than or equal to 9 congruent to 0 mod 9

$(10^m - 2)^2 = 10^{2m} - 4\cdot 10^m + 4$

$$= \underbrace{99\ldots9}_{m-1}6\underbrace{0\ldots0}_{m-1}4,\ m \ge 1.$$

The sum of the digits is $9m + 1$, which gives all values greater than or equal to 10 congruent to 1 mod 9.

$(10^m - 3)^2 = 10^{2m} - 6\cdot 10^m + 9$

$$= \underbrace{99\ldots9}_{m-1}4\underbrace{0\ \ldots\ 0}_{m-1}9,\ m \ge 1.$$

The sum of the digits is $9m + 4$, which takes every value greater than or equal to 13 which is congruent to 4 mod 9

$(10^m - 5)^2 = 10^{2m} - 10^{m+1} + 25$

$$= \underbrace{9\ \ldots\ 9}_{m-1}\underbrace{00\ldots0}_{m-1}25.$$

The sum of the digits is $9(m - 1) + 7 = 9m - 2$, from which we get every value greater than or equal to 7 congruent to 7 mod 9.

We have taken care of all the integers apart from 0, 1, 4, which are the sums of the digits of 0^2, 1^2 and 2^2 respectively.

5. Let O be the origin of a coordinate system where A, B, C, D are represented by $(a, 0)$, $(0, b)$, $(c, 0)$, $(0, d)$ with a, b positive and c, d negative. Thus L is the point

$\left(0, \dfrac{(b+d)}{2}\right)$, M is $\left(\dfrac{c}{2}, \dfrac{b}{2}\right)$, N is $\left(\dfrac{(a+c)}{2}, 0\right)$ and

$AL : (b + d)x + 2ay - a(b + d) = 0$

$OM : bx - cy = 0$

$DN : 2dx + (a + c)\,y - d(a + c) = 0$.

These lines are concurrent if and only if

$$\begin{vmatrix} b & -c & 0 \\ b+d & 2a & -a(b+d) \\ 2d & a+c & -d(a+c) \end{vmatrix} = 0.$$

This equation reduces (after some manipulation) to $(ab - cd)\,[(a - c)(b - d) + 2bc] = 0$.

Consequently, either

(a) $ab = cd$, in which case $AD \parallel BC$, or

(b) $\dfrac{1}{2}(a-c)(b-d)\sin\alpha = 2\left(-\dfrac{1}{2}bc\sin\alpha\right)$

(where $\alpha = \angle AOB$), in which case $[ABCD] = 2\,[OBC]$.

VIT Engineering Entrance Exam (VITEEE-2017) Results

VITEEE - 2017 TOP 10 RANK HOLDERS

RANK 1
AASHISH WAIKAR

RANK 2
DIVYANSH TRIPATHI

RANK 3
DIVYANSHU MANDOWARA

RANK 4
ABHISHEK RAO

RANK 5
BHANUTEJA BOLISETTI

RANK 6
HRITWIK SINGHAI

RANK 7
PRATHEEK D SOUZA REBELLO

RANK 8
AVVARI SAI S S V BHARADWAJ

RANK 9
PATEL MANAN BRIJESH

RANK 10
SHOURYA AGGARWAL

AASHISH WAIKAR, a student of BHAVAN VIDYALAYA PANCHKULA, Madhya Pradesh has secured the first rank in the VIT Engineering Entrance Examination (VITEEE)-2017 which was held from April 5^{th} to 16 in 119 selected cities across India, as well as Dubai, Kuwait and Muscat. The entrance exam was held for admission to the various B.Tech programmes offered by VIT University at its Vellore, Chennai, Bhopal & Amaravati (AP).

Releasing the results, VIT Chancellor Dr.G.Viswanathan said that a record 2,23,081 candidates had registered for the VITEEE-2017. The other rank holders among the top 10 are 2^{nd} rank: DIVYANSH TRIPATHI (Prabhat Sr Sec Public School, Uttar Pradesh), 3^{rd} rank: DIVYANSHU MANDOWARA (Arcadia Academy Co-Educational English Medium Senior Secondary School, Rajasthan), 4^{th} rank: ABHISHEK RAO (Remal Public School, Uttar Pradesh), 5^{th} rank : BHANUTEJA BOLISETTI (Sri Chaitanaya Narayana Jr College, Telengana), 6^{th} rank: HRITWIK SINGHAI (Little Kingdom Senior Secondary School, Madhya Pradesh), 7^{th} rank: PRATHEEK D SOUZA REBELLO (Mushtifund Aryaan Higher Secondary School, GOA), 8^{th} rank : AVVARI SAI S S V BHARADWAJ (Sri Chaitanya Junior Kalasala, Telengana), 9^{th} rank: PATEL MANAN BRIJESH (Shree Swaminarayan Secondary School, Gujarat) and 10^{th} rank : SHOURYA AGGARWAL (Hans Raj Model School, Delhi).

Dr. G. Viswanathan said that admissions would be only on merit, based on the marks obtained by the candidates in the VITEEE. The results have been released through the www.vit.ac.in.

Counselling for candidates, who obtained ranks upto 8,000 was held on May 10 and counseling for ranks 8001 to 14,000 was held on May 11 while for those who secured ranks from 14001 to 20000 was held on May 12. The counselling was held simultaneously in the Vellore, Chennai, Bhopal and Amaravati (AP).

Under the G V School Development Programme central and State board toppers would be given 100 percent fee waiver for all the four years. Candidates with ranks upto 50 would be given a 75% tuition fee waiver, Rank 51 to 100 would be given a 50% tuition fee waiver and Rank 101 to 1000 would be given a 25 % tuition fee waiver.

Each one boy and one girl secured top ranks in "Plus2" at district level from state board schools who also appeared for VIT Engineering Entrance Examination will be given 100% fee concession and free boarding and lodging in the hostels of VIT under STARS scheme.

☺☺

YOU ASK WE ANSWER

Do you have a question that you just can't get answered?
Use the vast expertise of our MTG team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough. The best questions and their solutions will be printed in this column each month.

1. For $r = 0, 1, \ldots, 10$, let A_r, B_r, C_r denote respectively, the coefficients of x^r in the expansion of $(1 + x)^{10}$, $(1+x)^{20}$, $(1+x)^{30}$. Then $\sum\limits_{r=1}^{10} A_r(B_{10}B_r - C_{10}A_r) =$

(Ram Krishan, West Bengal)

Ans. $(1 + x)^{10} = A_0 + A_1x + A_2x^2 + \ldots + A_{10}x^{10}$
$(x + 1)^{20} = B_0x^{20} + B_1x^{19} + B_2x^{18} + \ldots + B_{20}$.
Considering the coefficient of x^{20} in the product, we get $A_0B_0 + A_1B_1 + A_2B_2 + \ldots + A_{10}B_{10}$ = coefficient of x^{20} in the expansion of $(1 + x)^{10}(x + 1)^{20} = (1 + x)^{30}$ which is C_{20}

$$\therefore \quad \sum_{r=0}^{10} A_rB_r = C_{20} = C_{10}$$

$$\text{But } \binom{n}{0}^2 + \binom{n}{1}^2 + \binom{n}{2}^2 + \ldots + \binom{n}{n}^2 = \binom{2n}{n}$$

$$\therefore \quad A_0^2 + A_1^2 + A_2^2 + \ldots + A_n^2 = \binom{20}{10} = B_{10}$$

$$\sum_{r=0}^{10} A_r^2 = B_{10}. \text{ Now, } \sum_{r=1}^{10} A_r(B_{10}B_r - C_{10}A_r)$$

$$= B_{10}\sum_{r=1}^{10} A_rB_r - C_{10}\sum_{r=1}^{10} A_r^2$$

$$= B_{10}(C_{10} - 1) - C_{10}(B_{10} - 1) = C_{10} - B_{10}.$$

2. If $\begin{bmatrix} 4a^2 & 4a & 1 \\ 4b^2 & 4b & 1 \\ 4c^2 & 4c & 1 \end{bmatrix}\begin{bmatrix} f(-1) \\ f(1) \\ f(2) \end{bmatrix} = \begin{bmatrix} 3a^2+3a \\ 3b^2+3b \\ 3c^2+3c \end{bmatrix}$, $f(x)$ is a quadratic function and its maximum value occurs at a point V. A is a point of intersection of $y = f(x)$ with x-axis and point B is such that the chord AB subtends a right angle at V. Find the area enclosed by $y = f(x)$ and the chord AB.

(Suresh Prasad, Jharkhand)

Ans. The given equation implies, $(4f(-1) - 3)x^2 + (4f(1) - 3)x + f(2) = 0$ is satisfied by 3 roots a, b, c

$\Rightarrow$ It is an identity $\therefore f(-1) = \frac{3}{4}, f(1) = \frac{3}{4}, f(2) = 0$

If $f(\alpha) = \alpha x^2 + \beta x + \gamma$, then $\beta = 0$, $\alpha = -\frac{1}{4}$, $\gamma = 1$

$$\Rightarrow f(x) = -\frac{x^2}{4} + 1 = \frac{4 - x^2}{4}$$

The maximum point, $V = (0, 1)$; $A(-2, 0)$. Taking $B = (2t, 1 - t^2)$

$$\angle AVB = \frac{\pi}{2} \Rightarrow \frac{1}{2}\cdot\left(-\frac{t}{2}\right) = -1 \Rightarrow t = 4 \therefore B = (8, -15)$$

Equation of AB is $3x + 2y + 6 = 0$

$$\text{The area required is } \int_{-2}^{8}\left(\frac{4-x^2}{4} + \frac{3(x+2)}{2}\right)dx = 41.67$$

3. Let ABC be an equilateral triangle inscribed in the circle $x^2 + y^2 = a^2$. Suppose perpendiculars from A, B, C to the major axis meet the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $(a > b)$ respectively at P, Q, R so that P, Q, R lie on the same side of the major axis as A, B, C respectively. Prove that the normals to the ellipse at P, Q, R are concurrent.

(Priyanshu Sharma, Bihar)

Ans. A, B, C are the vertices of an equilateral triangle.
$A(a\cos\theta, a\sin\theta)$,

$$B\left(a\cos\left(\theta + \frac{2\pi}{3}\right), a\sin\left(\theta + \frac{2\pi}{3}\right)\right),$$

$$C\left(a\cos\left(\theta + \frac{4\pi}{3}\right), a\sin\left(\theta + \frac{4\pi}{3}\right)\right)$$

Hence, $P(a\cos\theta, b\sin\theta)$,

$$Q\left(a\cos\left(\theta + \frac{2\pi}{3}\right), b\sin\left(\theta + \frac{2\pi}{3}\right)\right),$$

$$R\left(a\cos\left(\theta + \frac{4\pi}{3}\right), b\sin\left(\theta + \frac{4\pi}{3}\right)\right)$$

The normals to the ellipse at P, Q, R are respectively,

$$L_1 \equiv ax\sin\theta - by\cos\theta - \frac{a^2 - b^2}{2}\sin 2\theta = 0;$$

$$L_2 \equiv ax\sin\left(\theta + \frac{2\pi}{3}\right) - by\cos\left(\theta + \frac{2\pi}{3}\right) - \frac{a^2 - b^2}{2}$$

$$\sin^2\left(\theta + \frac{2\pi}{3}\right) = 0; \; L_3 \equiv ax\sin\left(\theta + \frac{4\pi}{3}\right)$$

$$- by\cos\left(\theta + \frac{4\pi}{3}\right) - \frac{a^2 - b^2}{2}\sin 2\left(\theta + \frac{4\pi}{3}\right) = 0$$

$$\text{Since, } \sin\theta + \sin\left(\theta + \frac{2\pi}{3}\right) + \sin\left(\theta + \frac{4\pi}{3}\right) = 0$$

$$\cos\theta + \cos\left(\theta + \frac{2\pi}{3}\right) + \cos\left(\theta + \frac{4\pi}{3}\right) = 0$$

$$\sin 2\theta + \sin 2\left(\theta + \frac{2\pi}{3}\right) + \sin 2\left(\theta + \frac{4\pi}{3}\right) = 0$$

Hence L_1, L_2, L_3 are concurrent.

MATHS MUSING

SOLUTION SET-173

1. (c) :

Anil	Balu	Probability
R, R	R, R	1/33
B, B	B, B	1/33
R, B	R, B	10/33

$\therefore$ Required probability $= \frac{2}{33} + \frac{10}{33} = \frac{12}{33} = \frac{4}{11}$

2. (b) : Let (h, k) be the mid point of the chord of circle $x^2 + y^2 = a^2$, then the equation of the chord be $S_1 = T$

$\Rightarrow \; xh + yk - a^2 = h^2 + k^2 - a^2$

$\Rightarrow \; h^2 + k^2 = xh + yk$

$\Rightarrow \; h^2 + k^2 = ah + bk$ (As it passes through(a, b))

$\Rightarrow \; x^2 + y^2 = ax + by$ (By changing the locus of $h, k \to x, y$)

3. (b) : Let $I = \int_0^{\pi/2} \frac{dx}{\cos^6 x + \sin^6 x} \Rightarrow I = \int_0^{\pi/2} \frac{\sec^6 x}{1 + \tan^6 x} dx$

$$\Rightarrow I = \int_0^{\pi/2} \frac{(1+\tan^2 x)^3}{1+\tan^6 x} dx$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{1+\tan^6 x + 3\tan^2 x(1+\tan^2 x)}{1+\tan^6 x} dx$$

$$I = \int_0^{\pi/2} 1 + \frac{3\tan^2 x \sec^2 x \, dx}{1+(\tan^3 x)^2}$$

Put $\tan^3 x = t \;\Rightarrow\; dt = 3\tan^2 x \sec^2 x \, dx$

$$\Rightarrow I = \int_0^{\pi/2} dx + \int_0^{\infty} \frac{dt}{1+t^2} = \frac{\pi}{2} + \tan^{-1}(t)\Big|_0^{\infty} = \frac{\pi}{2} + \frac{\pi}{2} = \pi$$

4. (d) : Consider $C_r = \binom{24}{r}$

$(1 - x)^{24} = C_0 - C_1x + C_2x^2 - \ldots + C_{16}x^{16} + \ldots$

$(1 - x)^{-1} = 1 + x + x^2 + \ldots + x^{16} + \ldots$

Considering the coefficient of x^{16} in the product of these two,

$C_0 - C_1 + C_2 - \ldots + C_{16} =$ coeff. of x^{16} in $(1 - x)^{23}$

$$= \binom{23}{16} = \binom{23}{7}$$

5. (a) : Let the A.P.s. be $a, a + \alpha, a + 2\alpha, \ldots$ and $b, b + \beta, b + 2\beta, \ldots$

$\Rightarrow \; ab = a_1b_1 = 120, (a + \alpha)(b + \beta) = a_2b_2 = 143,$

$(a + 2\alpha)(b + 2\beta) = a_3b_3 = 154$

$\Rightarrow \; ab = 120, a\beta + b\alpha = 29, \alpha\beta = -6$

$\therefore \; a_8b_8 = (a + 7\alpha)(b + 7\beta) = ab + 7(a\beta + b\alpha) + 49\alpha\beta$

$= 120 + 7 \times 29 - 49 \times 6 = 29$

6. (b, c) : $f(x) = \log_e [x^3 + \sqrt{x^6 + 1}]$

$\therefore \; f(-x) = \log_e [-x^3 + \sqrt{x^6 + 1}]$

$\therefore \; f(x) + f(-x) = \log_e 1 = 0$

$f(-x) = -f(x)$

Hence $f(x)$ is an odd function.

$\therefore$ (b) is correct.

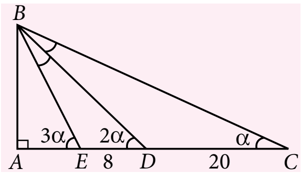
Again, $f'(x) = \frac{1}{x^3 + \sqrt{x^6+1}}\left[3x^2 + \frac{6x^5}{2\sqrt{x^6+1}}\right]$

$$= \frac{1}{x^3 + \sqrt{x^6+1}} 3x^2 \left[\frac{\sqrt{x^6+1} + x^3}{\sqrt{x^6+1}}\right] = \frac{3x^2}{\sqrt{x^6+1}} > 0$$

$\therefore f(x)$ is an increasing function, so (c) is correct.

7. (b) : $\angle DBC = \alpha \;\Rightarrow\; BD = 20, \; \angle EBD = \alpha$

By sine rule for ΔEBD, we get



$\frac{\sin 3\alpha}{20} = \frac{\sin \alpha}{8}$

$\Rightarrow \; 3 - 4\sin^2 \alpha = \frac{5}{2}$

$\Rightarrow \; \sin^2 \alpha = \frac{1}{8}, \; \cos 2\alpha = \frac{3}{4}.$

8. (d) : $AE + 28 = AB \cot \alpha$ and $AE + 8 = AB \cot 2\alpha$

On subtracting, we get $20 = AB (\cot \alpha - \cot 2\alpha)$

$\therefore \; AB = 20 \sin 2\alpha = 20\sqrt{1 - \frac{9}{16}} = 5\sqrt{7}$

$\therefore \; AE + 8 = AB \cot 2\alpha = 5\sqrt{7} \cdot \frac{3}{\sqrt{7}} = 15 \Rightarrow AE = 7.$

9. (8) : $a + b + c = 0 \Rightarrow a, b, c$ are the roots of $x^3 + qx + r = 0$ ...(i)

$\Sigma ab = q, abc = -r, a + b + c = 0$

$\Rightarrow \; \Sigma a^3 = 3abc \;\therefore\; abc = 1, r = -1, \Sigma a^2 = -2q$

From (i), we get $\Sigma a^5 + q\Sigma a^3 + r\Sigma a^2 = 0 \Rightarrow 10 + 3q + 2q = 0$

$\Rightarrow \; q = -2$

From (i), $x^3 - 2x - 1 = 0.$

$\therefore \; \Sigma a^3 - 2\Sigma a - \Sigma 1 = 0 \Rightarrow \Sigma a^4 = 2\Sigma a^2 + \Sigma a$

$\Sigma a^4 = 2(-2q) + 0 = (-4) \times (-2) = 8.$

10. (b) : P. $23^2 = -1 \bmod 53, 23^{22} = -1 \bmod 53,$

$23^{23} = -23 \bmod 53 = 30 \bmod 53$

Q. $X = x + 2, \; Y = y + 1, Z = z, U = u - 1$

$\Rightarrow \; X + Y + Z + U = 5$

$\therefore$ Number of solutions $= \binom{8}{3} = 56$

R. Coefficient of x^2y in $(1 + x + 2y)^5$ is $\frac{5!}{2! \cdot 2!}.2 = 60$

S. $\sum_{r=1}^{10} r\frac{(11-r)}{r} = 1 + 2 + \ldots + 10 = 55.$